Optimal Withdrawal Strategy for Retirement Income Portfolios

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1. Introduction

While a significant amount of research has been devoted to determining how much one can afford to withdraw from a retirement portfolio, surprisingly little work has been done on comparing the relative efficiency of different types of retirement withdrawal strategies. The purpose of this study is to first establish a framework to evaluate different withdrawal strategies and second to use that framework, in conjunction with Monte Carlo simulations[1], to determine the optimal withdrawal strategies for various case studies. To establish the framework, we introduce a new metric, the "Withdrawal Efficiency Rate" (WER), which measures the relative efficiency of various withdrawal strategies. The Withdrawal Efficiency Rate compares the withdrawals received by the retiree by following a specific strategy to what could have been obtained had the retiree had "perfect information" at the beginning of retirement. This measure allows us to quantify the relative appeal of each approach, and thus creates a framework to determine how best to generate income from a portfolio. Insofar as maximizing withdrawals, subject to a retiree's budget constraints, is a critical aspect of building a successful retirement plan, this framework should help both retirees and their advisors determine a more secure foundation for retirement spending. In particular, we will show that spending regimes that dynamically adjust for changes in both market and mortality uncertainties outperform the more traditional approaches.

The rest of paper is laid out in the following manner. In Section 2, we discuss previous work in this area and introduce the Withdrawal Efficiency Rate and the new evaluation framework. In Section 3, we analyze five different popular withdrawal strategies that are commonly used by financial planners by applying the withdrawal efficiency measure. In Section 4, we compare these five strategies and shed some light on the optimal withdrawal strategies for different types of investors. Section 5 consists of the conclusion and summary.

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[1] Monte Carlo is an analytical method used to simulate random returns of uncertain variables to obtain a range of possible outcomes. Such probabilistic simulation does not analyze specific security holdings, but instead analyzes the identified asset classes. The simulation generated is not a guarantee or projection of future results, but rather, a tool to identify a range of potential outcomes that could potentially be realized. The Monte Carlo simulation is hypothetical in nature and for illustrative purposes only. Results noted may vary with each use and over time.

2. Withdrawal Efficiency Rate

Most research on retirement portfolio withdrawal strategies has centered on the ability of a portfolio to maintain a constant withdrawal rate or constant dollar amount (either in real or in nominal terms) for some fixed period, such as 30 years. The annual withdrawal is commonly assumed to increase annually for inflation (we refer to this approach as "Constant Dollar" in this paper). Bengen (1994) is widely regarded as the first person to study the sustainable real withdrawal rates from a financial planning perspective. He found that a "first year withdrawal rate of 4%, followed by inflation adjusted withdrawals in subsequent years, should be safe." This is commonly referred to as the "4%" rule. Many experts and practitioners feel the 4% rule is rather naïve, as it ignores the dynamic nature of market and portfolio returns. More recent research has sought to determine the optimal withdrawal strategy by dynamically adjusting to market and portfolio conditions; for example, Guyton (2004), Guyton and Klinger (2006), Pye (2008), Stout (2008), Mitchell (2011), and Frank, Mitchell, and Blanchett (2011). These dynamic approaches can offer a more realistic path that retirees are more likely to follow since they continually "adapt" to the on-going returns of the portfolio. However, up until this point there has been no measure to evaluate the effectiveness of these withdrawal strategies (other than probability of failure, which has significant limitations).

Another common assumption in retirement research is the notion of a fixed retirement period, which is typically based on some percentile life expectancy. For example, if we have a male and female couple, both age 65, the probability of either (or both) members of the couple living past age 100 (35 years), based on the 2000 Annuity Mortality Table, is roughly 14%[2]. If 14% was determined to be an acceptable probability of outliving the retirement period for modeling purposes, 35 years would be selected as the retirement duration. The fixed-period approach essentially assumes retirees will live through the period without dying; i.e., this approach ignores another important dynamic retiree faces, the mortality probability. Assuming a fixed retirement period and then selecting a withdrawal rate based on that period is an incomplete methodology since this approach ignores the dynamic nature of mortality.

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[2] The probability of a 65-year-old male living to age 95 is 17%, the probability of a 65-year-old female living to age 95 is 23%, assuming independence, the probability of either member living to age 100 could be calculated: 1- ((100% - 17%)*(100% - 23%)) ≈ 14%.

Incorporating Perfect Information

Retirees face two unknowns when determining the best strategy to withdraw from a retirement portfolio to fund retirement: the future returns of the portfolio and the duration, or length, of the retirement period. If retirees knew the future return and the years they will live, i.e., if the retiree had "perfect information," he or she (or they for a couple) would be able to determine the precise amount of income that could be generated from the portfolio for life, eliminating any uncertainty about a shortfall (running out of money before death) or surplus (not spending all the money during the lifetime).

As we have shown in the preceding section, both constant withdrawal rate and fixed horizon planning—the most common approaches to assessing retirement withdrawal—leave out important aspects of what is relevant to a real failure or success of the retirement spending decision.[3] In general, determining the optimal withdrawal strategy is complicated since there are two unknown random variables (life expectancy and portfolio returns) that will have a dramatic effect on the potential income available. Because of this, no single comparison metric has emerged to compare the competing methodologies of the different strategies. This puts the retiree and a financial planner in a quandary, because there are a number of potential strategies retirees can choose among to draw retirement income. Common rules include "draw X% of your initial savings pool," "draw Y% of your current (i.e. constantly changing) account balance," or "draw the inverse of your life expectancy."[4]

This paper introduces a new measure called the "Withdrawal Efficiency Rate" (WER) that can be used to evaluate different withdrawal strategies and thus determine the optimal income-maximizing strategy for a retiree. The main idea behind WER is the calculation of how well, on average, a given withdrawal strategy compares with what the retiree(s) could have withdrawn if they possessed perfect information on both the market returns, including their sequencing, and the precise time of death. It is intuitively clear that, given a choice between two withdrawal strategies, the one that on average captures a higher percentage of what was feasible in a perfect-foresight world should be preferred.

To calculate the WER, we first need to calculate the Sustainable Spending Rate (SSR) under perfect information of market returns and life expectancy. (As indicated above, we use Monte Carlo simulations to generate both portfolio returns and the times of death.) For each simulation path the SSR is the maximum constant income a retiree could have realized from the portfolio had he or she (or they) known the duration of the retirement period and annual returns as they were to be experienced in retirement, such that it depletes the portfolio to zero at time of death. There is only one such number, and for a path of length N with market returns r_1,, r_2, …,r_N, the SSR, assuming the withdrawals are made at the start of each period, is given by the formula

$$SSR = \frac{1}{1 + \frac{1}{(1+r_1)} + \frac{1}{(1+r_1)(1+r_2)} + \ldots + \frac{1}{(1+r_1)(1+r_2)\ldots(1+r_{N-1})}}$$

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[3] The last one, fixed-horizon planning, is in effect the withdrawal formula the IRS mandates for Required Minimum Distributions, or RMDs, on tax-exempt savings accounts.

[4] More sophisticated approaches, as exemplified by Milevsky and Robinson 2005, incorporate the stochastic character of both the mortality and market returns, but are focused more on finding the "constant-dollar" probabilities of success or failure rather than finding the "best" strategy; the two are not equivalent. Milevsky's single exponential-mortality approximation is also not easily harnessed to work for couples.

(See Appendix 2 for the derivation. Since the withdrawals are made at the start of each year, the N-th year return does not enter into the formula.) The SSR is the numerator for the WER equation; it is the constant amount that it is feasible to withdraw for a given combination of market returns and death scenarios (we purposely disregard here the bequest motive, which in any case is secondary for most retirees). To calculate the numerator for the WER equation we need to first address the problem that most withdrawal strategies will produce cash flows that fluctuate through time. Even a "Constant Dollar" approach may be subject to one dramatic fluctuation when a retiree happens to outlive his or her assets. Therefore, for each series of potentially changing cash flows we calculate the "Certainty Equivalent Withdrawal" (CEW), based on a standard Constant Relative Risk Aversion (CRRA) utility function (we assume that the utility function is time separable, so that one can add the utilities of different-period cash flows):

$$u(C) = -\frac{C^{-\gamma}}{\gamma}$$

We assume a risk-aversion coefficient—gamma—of four to better reflect the risk-averse nature of the retirement planning where failure is penalized more heavily than success [5]. The CEW is the constant payment amount that a retiree would accept such that its utility (their sum, to be precise) would equal the utility of the actual cash flows realized on a given simulation path [6]. The sum of all the CEW payments is smaller than the sum of all the realized cash flows—by the nature of the CRRA utility function, a retiree would give up some of the potential cash flow amount to ensure a stream of unchanging cash flows. For a path of length N, with cash flows c_1, c_2, …,c_N, CEW is calculated form the formula below

$$N*(-\frac{CEW^{-\gamma}}{\gamma}) = \sum_{1}^{N} -\frac{c_i^{-\gamma}}{\gamma}$$

$$CEW = (\frac{1}{N}\gamma\sum_{1}^{N}\frac{c_i^{-\gamma}}{\gamma})^{-\frac{1}{\gamma}}$$

This process generates an equal-utility constant withdrawal amount for a given withdrawal strategy (even if the strategy involves non-constant cash flows), so the constant-amount equivalent of actual cash flows can be meaningfully compared against the constant cash flows achievable had the retiree had perfect information [7]. Therefore, the per-path Withdrawal Efficiency Rate (WER) can be expressed as:

$$WER = \frac{CEW}{SSR}$$

And the metric we are going to use is the average of per-path WERs.[8] The higher the average WER, the better the withdrawal strategy. We shall see that for plausible withdrawal strategies the average values of WER typically range between 50% and 80%.

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[5] It turns out that the results are not very sensitive to the precise choice of the risk-aversion coefficient.

[6] Williams and Finke (2011) also use the concept of Certainty Equivalent Withdrawal to assess the relative attractiveness of different withdrawal rates.

[7] Although the results would technically be the same if one just divided one utility by the other, the interpretation of the ratios of utilities would generally be very counterintuitive.

[8] In order to avoid infinitely negative utilities, which would result when the retiree(s) run out of money completely, we assume in our calculations that minimal payment or 0.1% of the initial portfolio value—which can be thought of as for example Social Security—is added each year to the payouts generated by the portfolio withdrawal strategy.

3. Analysis of Five Different Withdrawal Strategies

For the analysis, a Monte Carlo simulation is created where life expectancies and returns are randomized. Returns are based on a lognormal return distribution with market assumptions in Appendix I. The values are based on the historical returns of the Ibbotson Associates S&P 500 and US Intermediate Government Inflation Adjusted Total Return indexes. For conservative forecasting purposes, the portfolio return was reduced by 50 bps and the standard deviations were increased by 200 bps. Four equity allocations were considered for the analysis: 0% equities, 20% equities, 40% equities, and 60% equities, and 40% equities is considered for base case scenarios.

Life expectancies for males and females are based on the Annuity 2000 Mortality Table. The primary simulation will be based on the joint life expectancy of a couple, male and female, where the couple is assumed to be the same age (e.g., 65) and where the probability of each dying within a given year is independent. The retirement period is assumed to be "active" so long as either member of the couple (or potentially both) is still living.

Five different withdrawal strategies were reviewed for the analysis:

1. **Constant Dollar Amount:** Based on Initial Balance ("Constant Dollar")
   Withdrawal Amount: a fixed amount, increased annually by inflation, based on the initial balance at retirement

2. **Constant Percentage:** ("Endowment Approach")
   Withdrawal Amount: fixed percentage of portfolio value

3. **Changing Percentage** Probability of Failure Fixed Retirement Period ("Constant Failure Percentage")
   Withdrawal Amount: based on maintaining a constant probability of failure over the expected fixed retirement period

4. **Changing Percentage:** 1/Life Expectancy Withdrawal Approach ("RMD Method")
   Period Determination: updating based on survivorship experience
   Withdrawal Amount: 1 divided by the remaining retirement duration (life expectancy)

5. **Changing Percentage:** Probability of Failure Mortality Updating ("Mortality Updating Failure Percentage")
   Period Determination: updating based on survivorship experience
   Withdrawal Amount: based on maintaining a constant probability of failure over the estimated remaining retirement duration

The results of the WER approach will be reviewed independently for each of the five strategies for the base case scenario (65-year-old male and female joint couple), and then contrasted later in the paper. This provides the reader with information about not only what we believe is the best overall strategy, but the optimal withdrawal approach for each of the strategies reviewed.

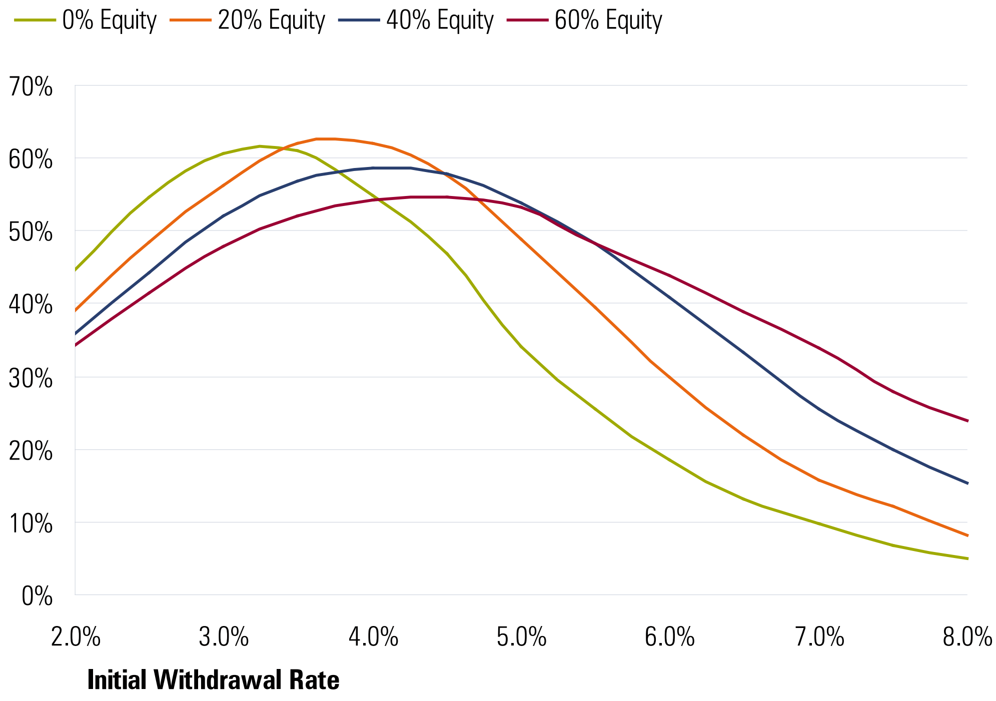
3.1 Constant Dollar

Early retirement research was typically based on a withdrawal rate that created a dollar amount that was assumed to be increased annually for inflation and withdrawn from the portfolio each year during retirement until the portfolio ran out of money. For example, a "4% Withdrawal Rate" would really mean a retiree can take a 4% withdrawal of the initial portfolio value and continuing withdrawing that amount each year, adjusted for inflation. If the initial portfolio value was $1 million, and the withdrawal rate was 4%, the retiree would be expected to generate $40,000 in the first year. If inflation during the first year was 3%, the actual cash flow amount in year two (in nominal terms) would be $41,200. Under this approach the withdraw amount is not related to the change in portfolio value or market return.

Figure 1 includes the Withdrawal Efficiency Rate (WER) obtained from Constant Dollar initial withdrawal rates for different equity allocations (0%, 20%, 40%, and 60%). As a reminder, in each case the WER is based on the cash flows the retiree actually obtained from the portfolio when compared against the cash flows that were available had that retiree had perfect information. The WER reflects the utility-adjusted income percentage of income received by the retiree (really retirees) versus the maximum potential income.

In Figure 1, note how the WER maximizing values were neither the most conservative nor the most aggressive initial withdrawal rates. This is because an initial withdrawal rate that is too conservative leaves too much potential income "on the table" that could have been spent during retirement, and an initial withdrawal rate that is aggressive results in the portfolio unable to generate income later in life. Given the utility function applied, running out of money is assigned a greater negative weight than not spending all available money, which is why the highest initial Constant Dollar withdrawals have the lowest WERs. The WER maximizing initial withdrawal rate was 3.5% for the 0% equity portfolio and 4.0% for the 20%, 40%, and 60% equity portfolios.

Figure 1: Withdrawal Rate Efficiencies for the Various Constant Dollar Approaches and Equity Allocations



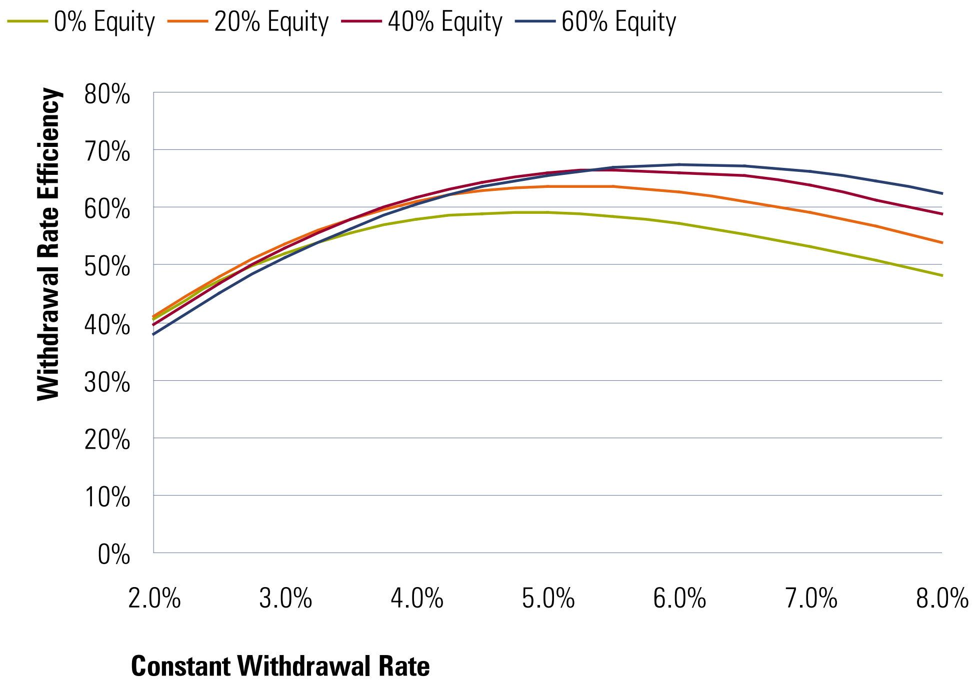
Source: Authors' calculations

An obvious concern with the Constant Dollar approach is that the cash flow is determined independently of the returns of the portfolio. In real life, if the portfolio experienced very poor returns initially the annual withdrawal would need to be reduced to ensure long-term survivability, and the reverse is true if the portfolio returns are good, whereby the amount of retirement income could be increased. These real life contingencies are not accounted for by the Constant Dollar strategy. This naturally suggests a withdrawal strategy of taking a perhaps time-varying percentage of the account balance every year as income, which ensures that the portfolio will not only never "fail," and which will make income "adapt" based on the performance of the portfolio. The remaining four strategies are based on withdrawing some percentage from the account through time.

3.2 The Endowment Approach (Constant Percentage)

Withdrawing a constant percentage of the account balance, named the Endowment approach, is perhaps the simplest approach a retiree can take with respect to withdrawing some percentage of the account balance. Under the Endowment approach, some constant percentage is withdrawn from the portfolio each year. Figure 2 contains the WERs for the four test equity allocations for various constant withdrawal percentages. The optimal withdrawal percentage was relatively constant across allocations, at 5.0% for the 0% and 20% equity allocations and 5.5% for the 40% and 60% equity allocations.

Figure 2: Withdrawal Rate Efficiencies for the Various Endowment Approaches and Equity Allocations



Source: Authors' calculations

The comparison of this chart with the preceding chart for constant dollar withdrawals makes it clear that the Endowment approach realized much higher values or WER across different withdrawal percentages and allocations when compared to the Constant Dollar approach.

3.3 Constant Failure Percentage

One method to help ensure portfolio sustainability is to determine the percentage that can be withdrawn each year based on the idea of maintaining a constant "probability of failure" (PoF) through time. With this approach, the withdrawal percentage is based on selecting the appropriate withdrawal percentage, based on what constant payment amount yields the target PoF. The goal is to maintain a constant PoF through time. To better help the reader understand this concept, the probabilities of failure for various time periods and equity allocations have been included in Table A.

Table A: Withdrawal Rates for Various Probabilities of Failure, Equity Allocations, and Time Periods

Probability of Failure at 0% Equity	5-Yrs	10-Yrs	15-Yrs	20-Yrs	25-Yrs	30-Yrs	35-Yrs	40-Yrs	45-Yrs
5%	18.3	9.0	5.9	4.5	3.6	3.0	2.7	2.4	2.1
10%	18.8	9.4	6.2	4.8	3.9	3.3	2.9	2.6	2.3
25%	19.7	10.0	6.8	5.3	4.3	3.7	3.3	3.0	2.7
50%	20.7	10.8	7.5	5.9	4.9	4.3	3.8	3.5	3.2

Probability of Failure at 20% Equity	5-Yrs	10-Yrs	15-Yrs	20-Yrs	25-Yrs	30-Yrs	35-Yrs	40-Yrs	45-Yrs
5%	18.5	9.2	6.2	4.8	3.9	3.4	3.0	2.8	2.5
10%	19.1	9.6	6.6	5.1	4.3	3.7	3.3	3.0	2.8
25%	20.0	10.4	7.3	5.7	4.8	4.2	3.8	3.5	3.3
50%	21.2	11.3	8.1	6.5	5.5	4.9	4.5	4.1	3.9

Probability of Failure at 40% Equity	5-Yrs	10-Yrs	15-Yrs	20-Yrs	25-Yrs	30-Yrs	35-Yrs	40-Yrs	45-Yrs
5%	17.8	8.9	6.0	4.7	3.9	3.4	3.1	2.8	2.7
10%	18.6	9.5	6.6	5.2	4.4	3.8	3.4	3.2	3.0
25%	19.9	10.5	7.5	5.9	5.1	4.5	4.1	3.9	3.7
50%	21.5	11.8	8.6	7.0	6.1	5.5	5.1	4.8	4.6

Probability of Failure at 60% Equity	5-Yrs	10-Yrs	15-Yrs	20-Yrs	25-Yrs	30-Yrs	35-Yrs	40-Yrs	45-Yrs
5%	17.1	8.4	5.6	4.4	3.6	3.2	2.8	2.6	2.4
10%	18.1	9.2	6.3	5.0	4.2	3.7	3.3	3.1	2.9
25%	19.8	10.6	7.5	6.0	5.2	4.7	4.3	4.0	3.9
50%	21.9	12.2	9.0	7.4	6.5	6.0	5.5	5.2	5.0

For example, if a retiree with a 40% equity allocation was interested in maintaining a 10% probability of failure and had a 30-year projected retirement period, the withdrawal percentage would be 3.8% in the first year (since there is 30 years remaining in the retirement period). In the 25th year the withdrawal percentage would be 4.4%, 5.2% in the 20th year, and 6.6% in the 15th year. Note how in Table B the withdrawal rates increase over shorter time periods and for higher failure probabilities. The higher the withdrawal amount, the higher the likelihood the portfolio will be unable to sustain for the time period, hence the higher failure rate. Table B can be built using the "Sustainable Spending Rate" methodology mentioned above, where the failure percentages represent percentiles from the distribution of all the paths' SSRs.

One obvious appeal of the Constant Failure Percentage approach is that it can work for a retiree regardless of how far the retiree is in retirement since it's a duration-based measure. The Constant Failure Percentage approach effectively creates a "distribution path" the retiree can follow each year with respect how much retirement income can be achieved from a portfolio. For example, we assume a retirement end age of 95, with a 40% equity portfolio and a 10% PoF, the 3.8% withdrawal rate would be the same for the 10th year of retirement for someone retiring at age 55, the 5th year of retirement for someone retiring at age 60, and be the current year withdrawal rate for someone who is 65.

The main problem with the Constant Failure Percentage approach is that it is not "mortality updating." If, for example, a retiree were to set the withdrawal period to 30 years and not update the period at all during retirement, the Constant Failure Percentage approach would mandate a 100% payout of the balance in the final (30th) year. Since this is a rather impractical assumption, for the purposes of the analysis the maximum withdrawal percentage is set to 25%. However, a 25% withdrawal would represent a significant withdrawal percentage for a couple who both manage to survive to age 90 and have a 34% chance of living at least another 10 years.

Table B includes the WERs for various equity allocations assuming a 34-year retirement period (living to age 99, which is based on the 20th percentile life expectancy) for the base case scenario for a Constant Failure Percentage approach. The corresponding first-year withdrawal percentage as a total of the account balance has also been included to give the reader an idea of the income generated from the portfolio in the first year. For each equity allocation the 50% PoF was optimal.

Table B: WERs for a Constant Failure Percentage Approach for Various Equity Allocations Assuming a 35-Year Retirement Period

	WERs Equity Allocation				**Corresponding First Year Withdrawal % Equity Allocation**			
Probability of Failure Target	0%	20%	40%	60%	0%	20%	40%	60%
5%	65.6	67.5	65.5	59.4	2.7	3.1	3.1	2.9
10%	67.5	69.2	68.6	64.9	2.9	3.4	3.5	3.4
25%	69.7	72.1	71.9	70.8	3.4	3.9	4.2	4.4
50%	70.7	72.7	73.0	72.0	3.9	4.5	5.1	5.6

3.4 The Required Minimum Distribution (RMD) Method

The key concern with the Constant Failure Percentage approach is that it is not "mortality updating," whereby the methodology does not incorporate the fact that the longer a retiree, or retiree couple, survive through retirement, the longer he or she (or they) are likely to live. One very simple method to incorporate mortality into a withdrawal rate methodology is to simply divide 1 by the remaining life expectancy. This approach is essentially the same methodology the IRS uses to establish the required minimum distribution (RMD) from a qualified plan by April 1 following the year an individual reaches age 70.

For example, based on the Annuity 2000 mortality values, the life expectancy of the base case couple, male and female both aged 65 is 28 years. Worded differently, the couple has approximately a 50% chance of living longer than 28 years at age 65. Therefore, under the RMD method, the annual withdrawal from the portfolio would be 3.6% (1/28=3.6%) for the first year of retirement (based on the 50% probability of outliving the distribution period). If a different probability were selected, e.g., a 10% probability of outliving the target horizon, the projected initial retirement period would be 37 years, which translates into a 2.7% initial withdrawal rate. Since the expected retirement period shortens every year a retiree (or retirees) survives, the withdrawal amount for the second year of the distribution period is going to be based on the now reduced life expectancy, and therefore will be a higher percentage of the portfolio (but potentially smaller account if portfolio value falls).

The simulation built to determine WER values randomly determines life expectancies based on the Annuity 2000 mortality values. Therefore, the simulation is able to "track" which members (both, one, or neither) of the original couple is alive and the corresponding remaining life expectancy (or retirement duration) based on the target probability of outliving the distribution period (the lower the probability, the longer the period). This is similar to how an actual retiree (or financial planner) would implement this approach to annually determine the sustainable withdrawal rate.

Table C: WERs for a RMD Approach for Various Equity Allocations for a 65-Year-Old Couple

	WERs **Equity Allocation**				**Corresponding First Year Withdrawal %** **Equity Allocation**			
Life Expectancy Target	0%	20%	40%	60%	0%	20%	40%	60%
5%	64.9	62.6	59.2	56.0	2.6	2.6	2.6	2.6
10%	68.1	66.0	62.5	59.2	2.7	2.7	2.7	2.7
25%	71.5	70.8	68.0	64.5	3.0	3.0	3.0	3.0
50%	66.4	68.3	67.4	66.4	3.6	3.6	3.6	3.6

Table D includes the WERs for various equity allocations and life expectancy targets based on the base scenario. Similar to Table C, the corresponding equity first-year withdrawal percentage as a total of the account balance has also been included to give the reader an idea of the income generated from the portfolio in the first year. The cash flows are based entirely on the distribution of death probabilities, which is why the first year withdrawal percentages don't change for different equity allocations. The 25% Life Expectancy Target was optimal for the 0%, 20%, and 40% equity allocations, while the 50% life expectancy target was optimal for the 60% equity allocation.

3.5 Mortality Updating Failure Percentage

The Constant Failure Percentage and RMD Method each have their own advantages. The Constant Failure Percentage allows a retiree to target a constant probability of failure (or success) for a given period, while the RMD method adjusts based on the remaining life expectancy. The final withdrawal strategy reviewed in this paper combines the Constant Failure Percentage approach and the RMD method, where the annual withdrawal is first based on the number of years remaining, then determined based on maintaining a constant probability of failure (PoF) for that period. This approach will be referred to as the "Mortality Updating Failure Percentage" approach.

The WER methodology is ideal for ranking the potential benefit of various Mortality Updating Failure Percentage approaches since there are two primary "levers" that can be adjusted for each given scenario. Given the specific age and desired portfolio allocation for a client, the "levers" are the target probability of failure and the target probability of outliving the distribution period. Both "levers" work in a similar manner, whereby using a more conservative assumption (lower probability of failure or lower probability of outliving the distribution period) reduces the initial cash flow from the portfolio. However, if a smaller amount is taken out of the portfolio initially, a larger amount can eventually be withdrawn based on portfolio survivability. The optimal approach, though, would be to balance these amounts over time.

Table D includes the results for the base scenario. The optimal combination (highest WER) value was based on a 25% life expectancy target for the 0% equity portfolios, but 10% for the 20%, 40%, and 60% equity portfolios. The target probability of failure was 50% for each of the four portfolio allocations. For simplicity purposes, and when taking into account secondary considerations like the average standard deviation of the change in cash flows, the 10% probability of failure target was determined to be the "global" optimal value for the Mortality Updating Failure Percentage approach.

Table D: Efficiency Rate (WER) for the Mortality Updating Failure Percentage Strategy

	WERs **Life Expectancy Target**				**Corresponding First-Year Withdrawal %** **Life Expectancy Target**			
Probability of Failure	0%	20%	40%	60%	0%	20%	40%	60%
5%	61.2	64.7	69.7	67.9	2.4	2.5	2.8	3.2
10%	64.6	67.5	71.4	68.5	2.6	2.7	3.0	3.5
25%	68.8	70.6	73.1	67.1	3.0	3.2	3.5	4.0
50%	71.4	72.7	73.1	64.3	3.5	3.7	4.0	4.5

	WERs **Life Expectancy Target**				**Corresponding First-Year Withdrawal %** **Life Expectancy Target**			
Probability of Failure	0%	20%	40%	60%	0%	20%	40%	60%
5%	64.3	66.6	71.7	69.7	2.8	2.9	3.2	3.6
10%	67.1	69.7	73.6	69.9	3.1	3.2	3.4	3.9
25%	71.8	73.6	75.3	68.9	3.6	3.7	4.0	4.4
50%	74.2	75.4	74.9	65.6	4.2	4.3	4.6	5.1

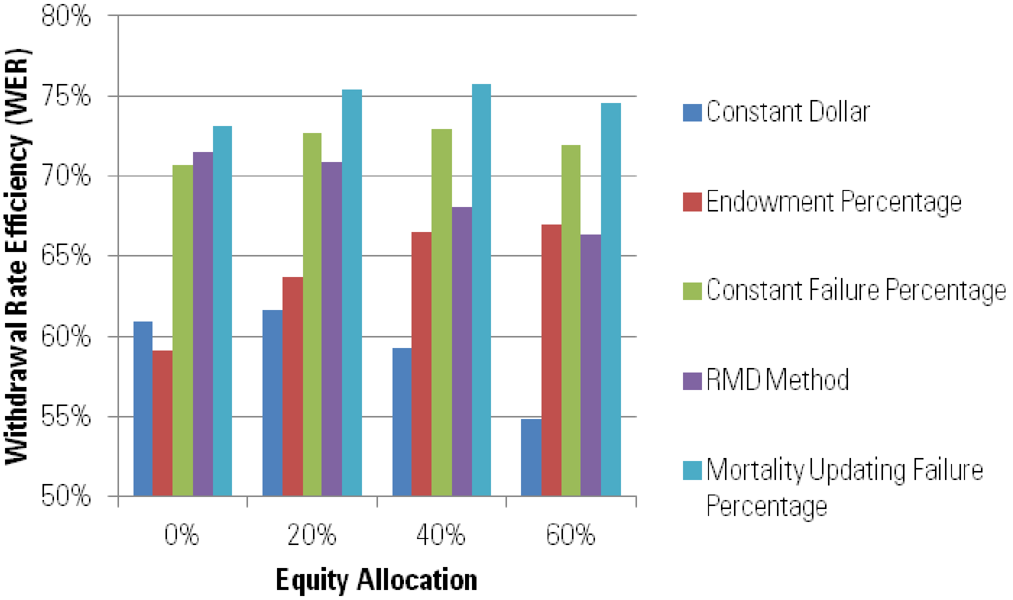
	WERs **Life Expectancy Target**				**Corresponding First-Year Withdrawal %** **Life Expectancy Target**			
Probability of Failure	0%	20%	40%	60%	0%	20%	40%	60%
5%	61.4	64.2	68.5	69.6	2.9	3.0	3.2	3.6
10%	65.9	68.0	72.2	70.3	3.2	3.3	3.6	4.0
25%	71.3	73.8	75.3%	69.9	3.9	4.0	4.3	4.7
50%	75.2	75.8	74.8	65.1	4.8	4.9	5.2	5.7

	WERs **Life Expectancy Target**				**Corresponding First-Year Withdrawal %** **Life Expectancy Target**			
Probability of Failure	0%	20%	40%	60%	0%	20%	40%	60%
5%	54.8	57.3	62.1	65.4	2.6	2.7	3.0	3.3
10%	61.2	63.5	67.7	68.5	3.1	3.2	3.4	3.8
25%	70.0	71.6	73.9	70.1	4.1	4.2	4.4	4.9
50%	74.1	74.6	74.0	0.0	5.3	5.4	5.7	0.0

4. Withdrawal Strategy Comparisons

Up to this point in the paper, each of the five withdrawal strategies have been reviewed individually and the optimal approach for that given strategy has been determined based on the WER value. Now we are going to contrast the relative efficiency of the optimal strategies among the five approaches. This information is included in Figure 3. Among the five withdrawal strategies considered and for each of the four different portfolios (equity allocations), the fifth strategy, Mortality Updating Failure Percentage, was the optimal withdrawal strategy; while the fourth strategy, Constant Failure Percentage approach, was the second best in three out of four equity allocations considered. For three out of four equity allocations, the Constant Dollar strategy was the worst. Also, interestingly, the Endowment Approach increases in relative efficiency for higher equity allocations, while the RMD method declines in relative efficiency.

Figure 3: Comparison of Five Withdrawal Strategies

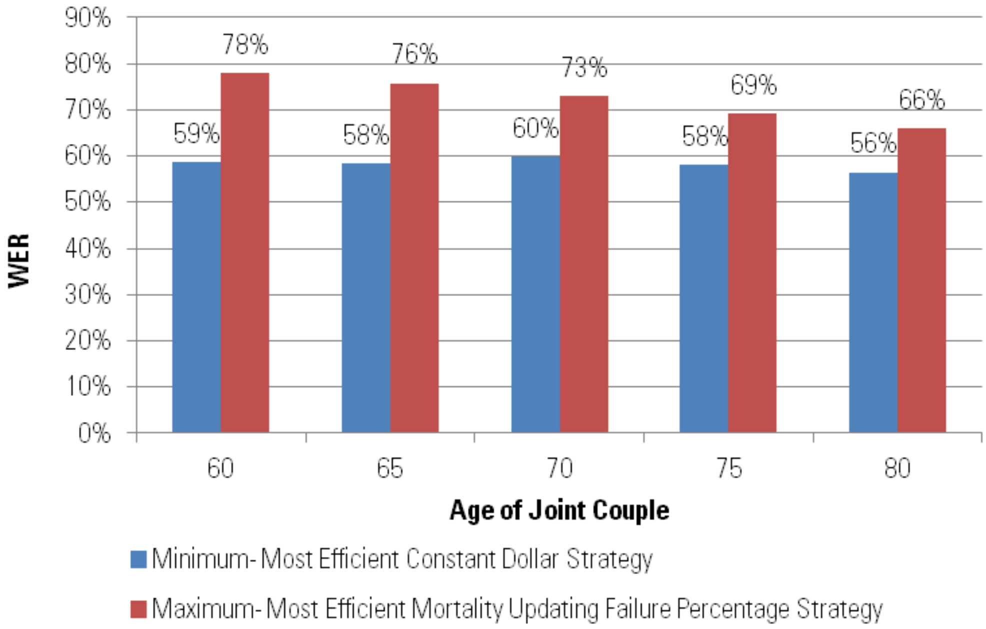


Source: Authors' calculations

The results make intuitive sense. Since market returns and mortality are stochastic variables, a probabilistic approach that incorporates the distribution of both into the withdrawal strategy (such as Mortality Updating Failure Percentage) should be expected to produce results that dominate strategies that focus on one, or none.

Additional Scenarios
The primary test case for this analysis was a joint couple, male and female, both age 65. Tests for other age combinations of a retired couple and for single retirees of various ages confirm this general ranking of different withdrawal regimes' efficiency. The Mortality Updating Failure Percentage approach was the most efficient approach for retirees ranging from age 60 to age 80, in five-year increments, for males, females, and joint couples (male and female assumed to be the same age). Note, though, the difference in the relative efficiency of the approaches decreased at older ages for joint couples, males, and females. Males and females tended to have lower efficiency scores when compared to joint couples. This relationship is depicted visually in Figure 4.

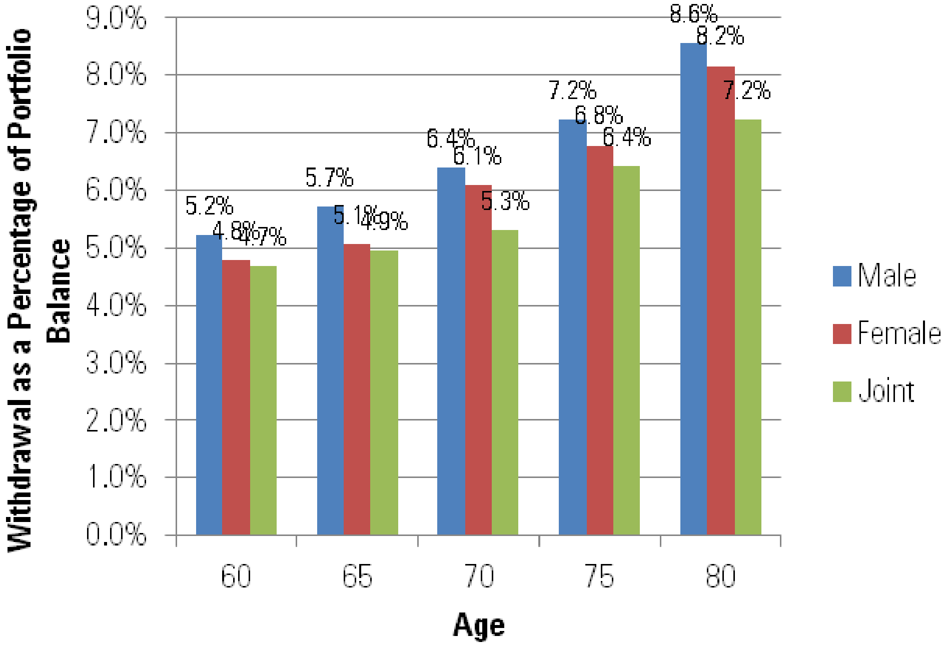
Figure 4: Comparison Difference in Optimal WER Values for Various Retirement Ages across the Five Strategies



Source: Authors' calculations

In order to provide general guidance as to what the target "levers" should be for the Mortality Updating Failure Percentage, we reviewed the results for the different test combinations (male/female/joint and ages 60/65/70/75/80). The general results suggest that a 50% would be the "global" optimal target probability of failure while a 10% probability of outliving the distribution period would work best for couples age 70 and under, while a 25% probability of outliving the distribution period would work best for couples 70 and older, as well as males and females. The probability of outliving the distribution period is higher for older couples, as well as males and females, given the shorter (on average) expected distribution period. The initial withdrawal percentages as a percentage of the portfolio balance for the 40% equity portfolios are included in Figure 5.

Figure 5: Withdrawal as a Percentage of Portfolio Balance for the Mortality Updating Failure Percentage Approach for a 40% Equity Portfolio



Source: Authors' calculations

5. Conclusion

This paper introduced a framework to determine the relative efficiency of different withdrawal strategies based on comparing the utility-adjusted cash flows against an income stream based on "perfect information." The measure, called the Withdrawal Efficiency Rate (WER) measures how big a percentage of what was feasible given perfect foresight a withdrawal strategy in question captures. We then empirically tested the WER across five withdrawal strategies through simulation analysis. The results suggest that the primary method employed by many practitioners, where a constant real dollar amount is withdrawn from the portfolio until it "fails" (called the "Constant Dollar" approach in this study) is often the least-efficient approach to maximizing lifetime income for a retiree.

The optimal withdrawal strategy points to approaches that incorporate mortality probability where the projected distribution period is updated based on the mortality experience of the retiree (or retirees) and the withdrawal percentage is determined based on maintaining constant probability of failure. This approach best replicates how a financial planner would (or at least should) determine the available income from a portfolio for each year during retirement. As a practical matter, for retirees who can't replicate the results presented here or don't have access to them, the RMD method emerges as a reasonable alternative to the more common constant dollar and constant percentage of assets withdrawal strategies.

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Appendix

Appendix 1: Market Forecasts (Real Values)

	Historical Values		Test Values	
Equity	Log Mean	Log St Dev	Log Mean	Log St Dev
0%	2.31	4.83	1.81	6.83
10%	2.89	4.94	2.39	6.94
20%	3.42	5.72	2.92	7.72
30%	3.92	6.96	3.42	8.96
40%	4.38	8.45	3.88	10.45
50%	4.81	10.09	4.31	12.09
60%	5.20	11.82	4.70	13.82
70%	5.55	13.60	5.05	15.60
80%	5.86	15.42	5.36	17.42
90%	6.14	17.27	5.64	19.27
100%	6.38	19.15	5.88	21.15

Appendix 2: Sustainable Spending Rate

Let's assume that you start with one dollar, and want to know how much you can spend per year in years 1 through N, if the initial dollar was invested in a portfolio with annual returns $r_1, r_2, \ldots, r_N$. We denote the spending rate by s.

You can spend s per year through year N is your final wealth at the end of year N is zero. This means solving the equation:

$$((1-s)(1+r_1)-s)(1+r_2)-s))\ldots(1+r_{N-1})-s=0$$

Continuing this way, we finally get to:

$$s=\frac{1}{1+\frac{1}{(1+r_1)}+\frac{1}{(1+r_1)(1+r_2)}+\ldots+\frac{1}{(1+r_1)(1+r_2)\ldots(1+r_{N-1})}}$$

Appendix 3: Probabilities of Survival for a Male and Female for Various Ages Based on the Annuity 2000 Mortality Table

	Male: Annuity 2000 Table Current Age					**Female: Annuity 2000 Table Current Age**					**Joint: Annuity 2000 Table Current Age**				
Death Age	60	65	70	75	80	60	65	70	75	80	60	65	70	75	80
65	96					98					100				
70	90	94				94	96				99	100			
75	81	84	90			88	90	94			98	98	99		
80	68	71	75	84		79	81	84	89		93	94	96	98	
85	51	53	56	63	75	64	65	68	72	81	82	84	86	90	95
90	32	33	36	40	47	43	44	46	49	55	62	63	65	69	76
95	16	17	18	20	23	22	23	24	25	29	35	36	37	40	45
100	6	6	6	7	8	8	9	9	9	11	14	14	15	16	18
105	1	1	1	1	2	2	2	2	2	2	3	3	3	4	4
110	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Source: Authors' calculations

Important Disclosures

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