Modern Portfolio Theory (MPT) Statistics

Morningstar Methodology Paper
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Introduction

Modern Portfolio Theory Statistics (MPT statistics) are based on the Capital Asset Pricing Model (CAPM) of expected returns developed by Nobel laureate William Sharpe and others in the early 1960s. The CAPM is based on Modern Portfolio Theory (MPT) developed in the 1950s by Sharpe's teacher and co-laureate Harry Markowitz.

In the terminology of another Nobel laureate, the late Milton Friedman, MPT is a normative theory, meaning that it is a prescription for how investors ought to behave. In contrast, the CAPM is a positive theory in that it meant to be a description of how investors do behave. The CAPM is based on MPT in that it assumes the all investors follow the prescriptions of MPT.

The CAPM separates the excess return (i.e., total return minus the return on a risk-free security) of each security into two components: systematic excess return and unsystematic (or idiosyncratic) return. Systematic excess return is directly proportional to the excess return of the market portfolio. The ratio of the excess return of the market to the systematic excess return of the security in question is the security's "beta." Beta measures how sensitive the excess return on a security is to the excess return of the market as a whole.

One of the main implications of the CAPM is that the expected excess return on a security is directly proportional to systematic risk as measured by beta and is not related to any other variable. This means that there are no rewards for taking on unsystematic risk. In an efficient market in which the CAPM holds, the only way to obtain an expected return above that of the market portfolio is to take on a beta above one.

In the 1970's, Michael Jensen proposed a performance measure for actively managed funds that is based on the CAPM called Jensen's alpha or simply alpha. The idea of alpha is that a manager should not receive credit for achieving above-market performance by taking on systematic risk as measured by beta. Alpha is the average excess return that a portfolio achieves above and beyond that could have been obtained from position in the market portfolio, levered or de-levered so as the have the same beta as the fund.

Introduction (continued)

Strictly speaking, the CAPM cannot be applied in the "real world" because returns on the market portfolio are unobservable. To make alpha and beta practical measures at first, broad stock market indexes were used as proxies for the market portfolio. As funds became more specialized, more narrow benchmarks were developed to track returns on the more narrowly defined sources of systematic risk and return. Today it is common practice to measure alpha and beta using a narrowly defined benchmark that is chosen to represent the main source of the systematic risk of the fund being analyzed.

In addition to alpha and beta, a third MPT statistic is R-squared. R-squared measures the strength of the relationship between excess returns on the benchmark and excess returns on the fund being analyzed.

MPT statistics are calculated from a comparison of a fund's excess returns and the benchmark's excess returns. Unless a time horizon is specified, Morningstar's MPT statistics are based on three years of monthly returns. Morningstar calculates MPT statistics for each fund, using a standard set of benchmarks for each asset group. Morningstar also calculates "best-fit" MPT statistics, which are based on the index that has the highest R-squared with the portfolio in question. For best-fit MPT statistics, Morningstar compares the portfolio to dozens of different indexes to find the best-fit.

Both the broad and best-fit results can be useful to investors. The broad index R-squared can help investors diversify their portfolios. For example, an investor who already owns a fund with a very high correlation (and thus high R-squared) with the S&P 500 might not choose to buy another fund that correlates closely to that index. The best-fit MPT statistics can help investors compare two similar funds. For example, if two funds have the same best-fit index, an investor can evaluate the risk and excess returns for those funds by comparing their best-fit betas and alphas.

Methodology

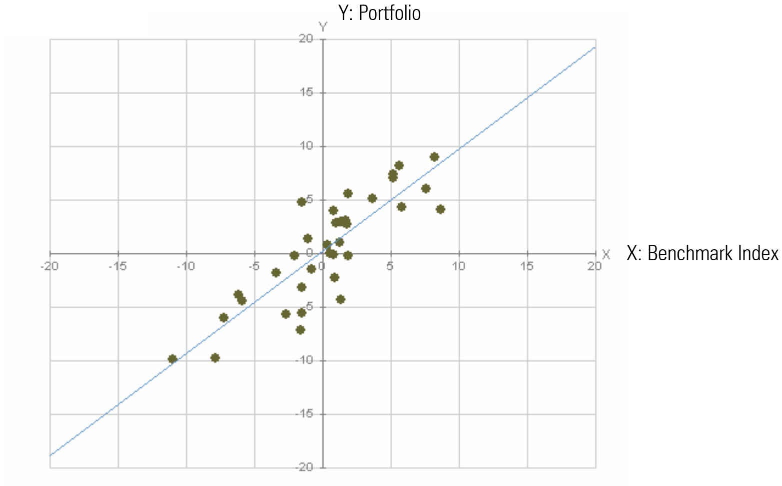
Morningstar calculates a fund's alpha, beta, and R-squared statistics by running *least-squares regression* of the fund's excess return over Treasury bills compared with the excess returns of the index that Morningstar has selected as the index for the fund's broad asset class. The Morningstar broad asset class indexes are as follows:

Broad Asset Class	Broad Asset Class Index
U.S. Stocks	S&P 500
International Stocks	MSCI EAFE
Balanced	Dow Jones Moderate Portfolio
Taxable Bonds	Lehman Brothers Aggregate Bond
Municipal Bonds	Lehman Brothers Muni

The calculations are made using the trailing 36-month period.

In least-squares regression there is *dependent variable* and an *independent variable*. A least squares regression can be understood by looking at a scatter plot of the independent variable on the horizontal axis and the dependent variable on the vertical axis. The *regression line* is the straight line that minimizes the sum of the squared vertical distances of each scatter point from the line.

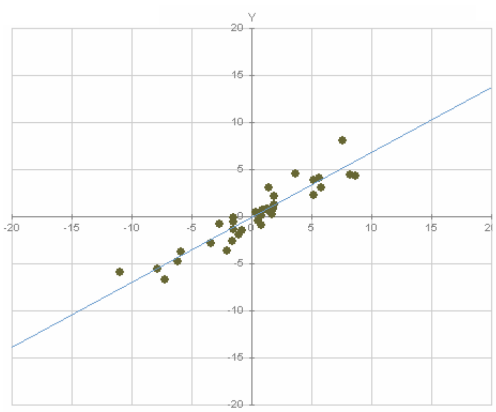
To estimate the MPT statistics for a fund using its broad asset class index, Morningstar runs a regression with the monthly excess return on the fund as the dependent variable and the excess return on the broad asset class index as the independent variable. The slope of the resulting regression line is the estimate of beta and the intercept (multiplied by 12 to express it as an annual figure) is the estimate of alpha.

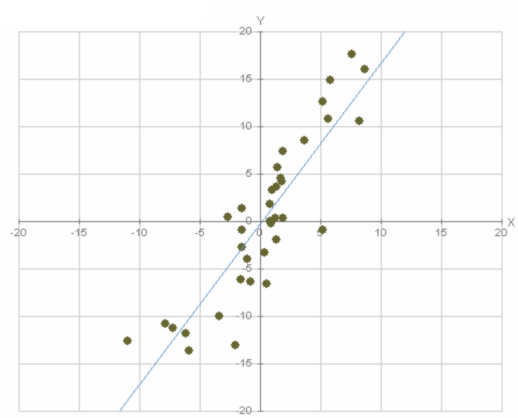


Beta

Beta is a measure of a fund’s sensitivity to movements in the index. By construction, the beta of the index is 1.00. A fund with a 1.10 beta has tended to have an excess return that is 10% higher than that of the index in up markets and 10% lower in down markets, holding all other factors remain constant. A beta of 0.85 would indicate that the fund has performed 15% worse than the index in up markets and 15% better in down markets. A low beta does not imply that the fund has a low level of volatility, though; rather, a low beta means only that the fund’s index-related risk is low. A specialty fund that invests primarily in gold, for example, will usually have a low beta (and a low R-squared), as its performance is tied more closely to the price of gold and gold-mining stocks than to the overall stock market. Thus, although the specialty fund might fluctuate wildly because of rapid changes in gold prices, its beta will be low.

In the examples below, beta is 0.69 for the fund on the left and 1.70 for the fund on the right.





Beta (continued)

Beta is calculated as:

$$\beta_{\mathrm{r}} = \frac{Cov_{rb}}{\sigma_{\mathrm{b}}^{2}}$$

where:

β_r	=	Beta of portfolio r
Cov_{rb}	=	Covariance between the excess returns of the portfolio r and the benchmark b
σ_b^2	=	Variance of the excess returns of the benchmark

and:

$$Cov_{\mathrm{rb}} = \frac{1}{\mathrm{n}-1}\sum_{\mathrm{i}=1}^{\mathrm{n}}[(R_i^e - \overline{R}^e)(B_i^e - \overline{B}^e)]$$

where:

R_i^e	=	Excess return of the portfolio for month i = R_i - RF_i, where R_i is the portfolio return for month i and RF_i is the risk-free return for month i
$\overline{R}^e$	=	Average monthly excess return of the portfolio over n periods (simple mean)
B_i^e	=	Excess return of the benchmark for month i = B_i - RF_i, where B_i is the benchmark return for month i and RF_i is the risk-free return for month i
$\overline{B}^e$	=	Average monthly excess return of the benchmark index over n periods (simple mean)
n	=	number of periods (Morningstar typically uses 36 months)

$\overline{R}^e$ is the simple arithmetic average excess return for the portfolio:

$$\overline{R}^e = \frac{1}{n}\sum_{i=1}^{n} R_i^e$$

The denominator for beta is the variance of the excess returns of the benchmark:

$$\sigma_b^2 = \frac{1}{\mathrm{n}-1}\sum_{\mathrm{i}=1}^{\mathrm{n}}(B_i^e - \overline{B}^e)^2$$

A similar calculation can also be used for the variance of the portfolio, σ_r^2.
Standard deviation is the square root of variance.

Alpha

Alpha measures a fund's performance after adjusting for the funds systematic risk as measured by the fund's beta with respect to the index. An investor could have formed a passive portfolio with the same beta that of the fund by investing in the index and either borrowing or lending at the risk-free rate of return. Alpha is the difference between the average excess return on the fund and the average excess return on the levered or de-levered index portfolio. For example, if the fund had an average excess return of 6% per year and its beta with respect to the S&P 500 was 0.8 over a period when the S&P 500's average excess return was 7%, its alpha would be 6% – 0.8*7% = 0.4%.

There are limitations to alpha's ability to accurately depict a fund's added or subtracted value. In some cases, a negative alpha can result from the expenses that are present in the fund figures but are not present in the figures of the comparison index. The usefulness of alpha is completely dependent on the accuracy of beta. If the investor accepts beta as a conclusive definition of risk, a positive alpha would be a conclusive indicator of good fund performance.

$$\alpha_M = \overline{R}^e - \beta \overline{B}^e$$

where:

α_M	=	Monthly measure of alpha
$\overline{R}^e$	=	Average monthly excess return of the portfolio
$\overline{B}^e$	=	Average monthly excess return of the benchmark index

The resulting alpha is in monthly terms, because the average returns for the portfolio and benchmark were monthly averages. Morningstar then annualizes alpha to put it in annual terms.[1]

$$\alpha_A = 12\,\alpha_M$$

where:

α_A	=	Annualized measure of alpha

[1] Prior to 2/28/2005, Morningstar annualized the monthly alpha with a geometric method, $(1+\alpha)^{12}-1$.

R-squared

R-squared is another statistic that is produced by a least-squares regression analysis. R-squared is a number between 0 and 100% that measures the strength of the relationship between the dependent and independent variables. An R-squared of 0 means that there is no relationship between the two variables and an R-squared of 100% means that the relationship is perfect with every scatter point falling exactly on the regression line. Thus, stock index funds that track the S&P 500 index will have an R-squared very close to 100%.

A low R-squared indicates that the fund's movements are not well explained by movements in the index. An R-squared measure of 35%, for example, means that only 35% of the fund's movements can be explained by movements in the index.

R-squared can be used to ascertain the significance of a particular beta estimate. Generally, a high R-squared will indicate a more reliable beta figure.

R-squared ranges from 0 (perfectly uncorrelated) to 100 (perfectly correlated). Correlation (ρ) is the square root of R-squared. R-squared is calculated as follows:

$$R^2 = 100 \left(\frac{Cov_{\mathrm{rb}}}{\sigma_{\mathrm{r}}\sigma_{\mathrm{b}}}\right)^2$$

where:

Cov_{rb}	=	Covariance between the excess returns of the portfolio r and the benchmark index b
σ_{r}	=	Standard deviation of the excess returns of the portfolio r
σ_{b}	=	Standard deviation of the excess returns of the benchmark index b

Best-Fit Index

Morningstar also shows additional alpha, beta, and R-squared statistics based on a regression against the best-fit index. The best-fit index for each fund is selected based on the highest R-squared result from separate regressions on a number of indexes. For example, many high-yield funds show low R-squared results and thus a low degree of correlation when regressed against the broad asset class index for the taxable bond funds, the Lehman Brothers Aggregate. These low R-squared results indicate that the index does not explain well the behavior of most high-yield funds. Most high-yield funds, however, show significantly higher R-squared results when regressed against the CSFB High-Yield Bond index.

Both the broad asset class and best-fit results can be useful. The broad asset class index R-squared statistics can help plan the diversification of a portfolio of funds. For example, if an investor wishes to diversify and already owns a fund with a very high correlation (and thus a high R-squared) with the S&P 500, he or she might choose not to buy another fund that correlates closely to that index. In addition, the best-fit index can be used to compare the betas and alphas of similar funds that show the same best-fit index.