Morningstar Quantitative Risk Ranking
Methodology

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Content

Michael Manetta, CFA
Lead Quantitative Analyst
+1 312-244-7677
michael.manetta@morningstar.com

Introduction

Global wealth has increased dramatically over the past several decades, both in the aggregate and on a per capita basis. At the same time, capital markets have continued to broaden and deepen around the world, giving more people access to a wider selection of investment products in which to invest that wealth.

The increased availability of investment vehicles and their growing complexity have spurred regulators in many jurisdictions to take a more systematic approach to suitability requirements. As distributors of investment products, wealth managers are under increasing pressure to refine their product risk rating framework and ensure that the inherent risks in such products are effectively captured and matched to their clients' risk profiles and tolerances.

Morningstar aims to meet this growing regulatory demand by providing a method of ranking the universe of funds available for sale in a target market or regulatory jurisdiction. Unlike a risk rating, which is typically based on absolute measures of risk, this ranking scheme is designed to provide regulators and, by extension, advisors and end-investors a consistent and comprehensive way to evaluate a product's riskiness relative to other available investment products. The initial ranking system is a simple 1-5 scale representing an ascending order of riskiness--that is, funds ranked 1 are the least risky, and funds ranked 5 are the riskiest. Rankings are specific to the target market, meaning the same product can have a different ranking in different jurisdictions.

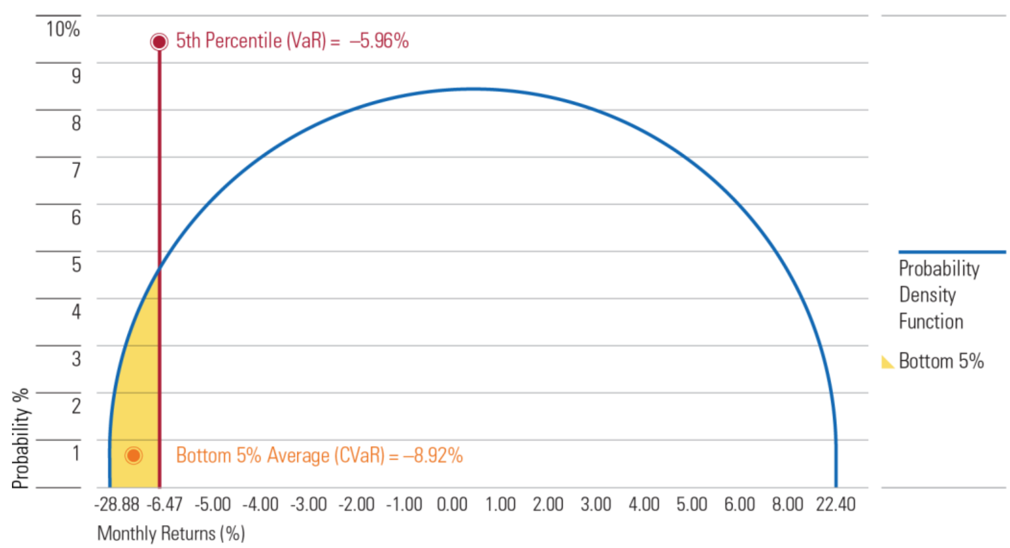
Overview of the Risk Ranking Methodology

Risk rankings are determined by calculating a conditional value-at-risk, or CVaR, for each fund available for sale in a target market, then assigning a risk ranking to each fund based on the relative order of CVaRs from least risky to most risky.

CVaR--also known as expected shortfall, mean shortfall, or mean excess loss--measures the expected loss on an investment assuming losses exceed some predetermined threshold. Put differently, CVaR answers the question, "How much should I expect to lose on an investment assuming I lose at least X." "X" can stand for a variety of values; for the purposes of the quantitative risk ranking, X is defined as the 5th percentile of the historical return distribution of a fund. We can thus rewrite the question as, "What is the average of the bottom 5% of an investment's historical returns?"

CVaR is related to the concept of value-at-risk, or VaR, which is defined as the lowest potential loss on an investment given a probability. To illustrate these concepts, Exhibit 1 shows the probability density function, or PDF, for the historical returns of Fidelity Contrafund FCNTX, an open-end mutual fund of U.S. large-cap equities. The VaR tells us what the maximum expected loss on this fund would be 95% of the time, based on historical returns. In this case, the maximum loss is 5.96%. The CVaR tells us what the expected loss would be for the remaining 5% of the time, that is, the average loss in the bottom 5% of the historical return distribution, in this case a loss of 8.92%.

Exhibit 1 Probability Density Function of Historical Monthly Returns for Fidelity Contrafund



Source: Morningstar. Data as of July 31, 2019.

Fidelity Contrafund has return history going back to 1967, such that the bottom 5th percentile of monthly returns has more than 30 observations. As such, the maximum loss of 28.88% from October 1987 does not overly influence the historical CVaR value of a loss of 8.92%. In general, the longer the return history, the more confident we can be in our CVaR estimates. Unfortunately, fund return histories can vary considerably, even within a single target market. Newer funds with a dearth of return history present a particularly difficult challenge in this regard.

To better align the return histories of the funds available for sale in a target market, we construct "synthetic" return series for each fund by employing a linear regression technique known as returns-based style analysis, or RBSA. RBSA maps the returns of a fund to the returns of a group of indexes that have a longer return history than the fund itself. In addition, the RBSA approach ensures that each fund's synthetic return series has the same length and covers the same time period. In this way, the CVaRs calculated from these synthetic returns will reflect the same market regimes and systemic shocks, rendering the CVaRs more readily comparable.

Take the following stylized example:

$$r_{Fund\ A} = \beta_1 * (r_{Index\ 1}) + \beta_2 * (r_{Index\ 2}) + \beta_3 * (r_{Index\ 3}) + \beta_4 * (r_{Index\ 4})$$

Where:
$r_{Fund\ A}$ = the monthly total return of Fund A;

$\mathrm{r}_{\mathrm{Index}\ 1-4}$ = the monthly total return of Indexes 1-4;

And $\beta_1 + \beta_2 + \beta_3 + \beta_4 = 1$

The RBSA regression estimates coefficients for Indexes 1-4, with the constraint that the coefficients sum to 1. Indexes with returns that closely align with a fund's returns are given a larger weight.

One challenge with the RBSA approach is determining which indexes to use on the right-hand side of the regression. Selecting the appropriate indexes for a single fund may be a reasonable task: For instance, to evaluate an international equity fund, one might select five regional or country-specific equity indexes, one or two commodity indexes, a global high-yield bond index, and a U.S. Treasury index. To evaluate a European corporate bond fund, a different set of indexes would probably be more appropriate. As more funds are added to the analysis, more unique sets of indexes are potentially needed to deliver reliable estimates.

There is also the question of how many indexes to use in the regression. A more parsimonious model runs the risk of excluding indexes that have a stronger statistical relationship to the fund in question; too many right-hand-side variables risks overfitting.

We resolve these issues by utilizing a least absolute shrinkage and selection operator, or LASSO, to mimic the output of RBSA while also performing index selection. Like ordinary least squares, or OLS, LASSO finds the coefficients that minimize the squared difference between the model-derived fitted values and the actual observation values. In minimizing these error terms, LASSO adds a "penalty" term that increases in tandem with the absolute size of the estimated coefficients. This forces the optimization process to reduce the size of the coefficient beyond what it would be under a normal OLS model. At a certain threshold point, LASSO resorts to setting the coefficient to zero, effectively eliminating it from the model (see Appendix B).

We continue the stylized example above to illustrate how this process works and how it can be used to produce RBSA-type output. Assume the model represented by the equation above using OLS to get the following estimated coefficients:

$$r_{Fund\ A} = \mathbf{0.25} * (r_{Index\ 1}) + \mathbf{0.02} * (r_{Index\ 2}) + \mathbf{0.20} * (r_{Index\ 3}) + \mathbf{0.35} * (r_{Index\ 4})$$

Note that the estimated coefficient for Index 2 is relatively small compared with the other estimated coefficients. LASSO adds a large penalization parameter to the error term and "shrinks" the coefficient to a point where the optimization process assigns a zero coefficient:

$$r_{Fund\ A} = \mathbf{0.25} * (r_{Index\ 1}) + \mathbf{0.00} * (r_{Index\ 2}) + \mathbf{0.20} * (r_{Index\ 3}) + \mathbf{0.35} * (r_{Index\ 4})$$

Next, we divide each LASSO-derived coefficient by the sum of all the LASSO-derived coefficients to calculate the RBSA-type weights for each index:

$$\hat{\beta}_{Total} = \underbrace{0.25}_{\hat{\beta}_1} + \underbrace{0.00}_{\hat{\beta}_2} + \underbrace{0.20}_{\hat{\beta}_3} + \underbrace{0.35}_{\hat{\beta}_4} = 0.80$$

And:

$$w_1 = \frac{\hat{\beta}_1}{\hat{\beta}_{Total}} = \frac{0.25}{0.80} = \mathbf{0.31}$$

$$w_2 = \frac{\hat{\beta}_2}{\hat{\beta}_{Total}} = \frac{0.00}{0.80} = \mathbf{0.00}$$

$$w_3 = \frac{\hat{\beta}_3}{\hat{\beta}_{Total}} = \frac{0.20}{0.80} = \mathbf{0.25}$$

$$w_4 = \frac{\hat{\beta}_4}{\hat{\beta}_{Total}} = \frac{0.35}{0.80} = \mathbf{0.44}$$

We calculate the synthetic fund return series as the weighted sum of the index returns, using w_1, w_2, w_3, w_4 as the weights:

$$r(syn)_{Fund\ A} = 0.31 * (r_{Index\ 1}) + 0.00 * (r_{Index\ 2}) + 0.25 * (r_{Index\ 3}) + 0.44 * (r_{Index\ 4})$$

Where:

$r(syn)_{Fund\ A}$ = the synthetic monthly total return of Fund A.

Risk Rankings Using Returns-Based Style Analysis

RBSA is a parametric statistical method originally devised to evaluate a fund manager's investment "style." A fund's return history is regressed on the returns of a predetermined set of indexes, where each index represents a different asset class and/or investment approach. The estimated coefficients of the regression are constrained to sum to 1, and the coefficients are interpreted as "weights" indicating which indexes--and thus which asset classes and investment strategies--best explain the strategy or style of the fund manager.

The RBSA regression:

$$r_{f,t} = \sum_{i=1}^{I} \beta_i * r_{i,t}$$

Subject to:

$$\sum_{i=1}^{I} \beta_i = 1$$

Where:

$r_{f,t}$ = Return of fund f at time t

$r_{i,t}$ = Return of index i at time t

I = Set of indexes i

Synthetic Fund Returns

The estimated coefficient on each index can also be used as a weight to construct a synthetic return history for a fund, which can be useful when a fund has a short return history. For example, it is difficult to assess the risk/reward characteristics of a fund with only four years of monthly return history. To get a longer time series, one can regress the fund's 48 months of returns on a curated set of index returns with (presumably) much longer return histories. The estimated coefficients from this regression--constrained to sum to 1--can then be used to construct a much longer "synthetic" fund return series as a weighted sum of the index returns.

Index Selection

Indexes are selected according to several criteria:

- Universe coverage: Ideally, each index in the set would be a near-mutually exclusive representation of a particular asset class, sector, industry, and region. As such, region-, country-, and sector-specific indexes for equity, rates, credit, and alternatives are selected to capture as much of the investable universe as possible.

- Return history: The indexes should have sufficiently long return histories such that the synthetic fund returns derived from the RBSA process include multiple business cycles. In practice, we select indexes that have return histories going back to January 2003 (or earlier).

Using these filters, we selected approximately 280 indexes to represent the investable universe. See Appendix A for the global index list.

Definitions and Naming Conventions

- **Target Market**: The country or regulatory jurisdiction for which risk rankings are being assigned. The target market currency is local currency of the target market, such as the euro for Germany, the British pound for the U.K., and so on.

- **Available for Sale (AFS) Universe**: The share classes representing the entire investable universe of managed products in a target market. The AFS universe is defined at the share class level, meaning the same fund may have multiple share classes available for sale in the target market. The AFS currency is the currency in which the returns of the AFS share class are denominated.

- **Oldest Share Class (OSC)**: The share class with the longest active return history in a fund. Often there are multiple "oldest" share classes with the same length of return history. The OSC currency is the currency in which the returns of the OSC are denominated. It may differ from the AFS currency.

- **Category Average (CA):** Morningstar Category averages are designed to represent the average return of funds within their category over time. It will be structurally different from the mean return of the current constituents of the category as it will take into account funds that have changed categories over time and share classes/funds that have subsequently liquidated. This ensures that the category averages are free of survivorship bias. The CA currency is the currency in which the CA returns are denominated.

- **Category Benchmark (CB)**: Primary and secondary indexes are assigned to each Category and are used in Morningstar’s tools and reports to show performance relative to a benchmark. The CB will also have its own currency.

The Quantitative Risk Ranking Process

Below is the order of operations by which risk rankings are assigned to each fund available for sale in a target country:

1) Ensure a Sufficient Return History to Calculate CVaR

Calculating a returns-based CVaR requires a return history of sufficient length to identify the moments of the return distribution with a reasonable amount of confidence. What constitutes "sufficient length" is somewhat subjective and context-specific; in our view, a return series going back to January 2003 is sufficient from both a statistical perspective (January 2003 to December 2019 constitutes 204 return observations) and a business cycle perspective (we are capturing two broad economic expansions, one large contraction, and multiple "mini-cycles" such as the 2015-16 credit sell-off).

For more mature funds, we can use the actual return histories back to 2003 to calculate CVaR (detailed below). For younger funds, we employ a LASSO-RBSA approach to create a "synthetic" return history going back to 2003. To supplement the fund-level CVaR, we use the same logic to calculate a Category Average and Category Benchmark CVaR as well.

- Convert the Oldest Share Class (OSC), Category Average (CA), Category Benchmark (CB), and Global Index (GI) returns to the target market currency tmc:

$$\boldsymbol{R}_f^{tmc} = \left[\left(1 + \boldsymbol{R}_f^{fc}\right) * \left(1 + \boldsymbol{R}_{tmc}^{fc}\right) - 1\right] \forall\, f, fc \in (OSC, CA, CB, GI)$$

Where:

$\boldsymbol{R}_f^{tmc}$ = A matrix of returns for $f \in (OSC, CA, CB, GI)$ denominated in tmc;

$\boldsymbol{R}_f^{fc}$ = A matrix of returns for f denominated in $f's$ corresponding currency, fc;

$\boldsymbol{R}_{tmc}^{fc}$ = A vector of returns of currency tmc relative to currency fc.

- Regress each return series for OSC, CA, and CB on the set of GI returns using a LASSO. The LASSO acts as a variable selection tool by "shrinking" to zero the coefficients for indexes exhibiting multicollinearity (redundancy) and/or low economic significance.

  For long-only funds, category averages, and category benchmarks, the estimated coefficients are constrained to be positive. For funds that allow shorting, the estimated coefficients can be positive or negative.

$$\widehat{\boldsymbol{R}}_{f \in (OSC,CA,CB)}^{tmc} = \boldsymbol{R}_{f=GI}^{tmc} \cdot \widehat{\boldsymbol{\beta}}_f$$

Which returns:

$\widehat{\boldsymbol{\beta}}_f$ = A matrix of coefficients estimated from the LASSO regression of f;

$\boldsymbol{R}_{f=GI}^{tmc}$ = A matrix of index returns associated with estimated coefficients of f.

See Appendix B for additional details on LASSO regressions.

- Recalculate the k nonzero index coefficients so that they collectively sum to 1;

$$\widetilde{\boldsymbol{\beta}}_{f(k)} = \frac{\widehat{\boldsymbol{\beta}}_f}{\sum_{k=1}^{K} \widehat{\boldsymbol{\beta}}_f}$$

- Use the recalculated coefficients in $\widetilde{\boldsymbol{\beta}}_{\mathrm{f(k)}}$ as weights to construct a synthetic return series for each f by taking the dot product of the returns and weights at each time *t:*

$$\widetilde{\boldsymbol{R}}_{f\in(OSC,CA,CB)}^{tmc} = \boldsymbol{R}_{f=GI(k)}^{tmc} \cdot \widetilde{\boldsymbol{\beta}}_{\mathrm{f(k)}}$$

Where:
$\widetilde{\boldsymbol{R}}_{f\in(OSC,CA,CB)}^{tmc}$ = the synthetic return series for f in tmc;

$\boldsymbol{R}_{f=GI(k)}^{tmc}$ = a vector of returns for the k indexes returned from the LASSO regression.

Note that we generate LASSO-RBSA-derived weights for as many OSC returns, CA returns, and CB returns as possible. However, we only construct synthetic returns from these weights in cases where the underlying return series does not go back to January 2003. Otherwise, in what follows, we use the actual returns from the OSC, CA, or CB, denominated in the target currency.

2) Calculate CVaR by Sampling from Bivariate Normal Distribution

- Convert the OSC, CA, and CB returns--currently denominated in the target market currency--to their corresponding AFS, CA, and CB currencies.

$$\widetilde{\boldsymbol{R}}_f^{fc} = \left[\left(1 + \widetilde{\boldsymbol{R}}_f^{tmc}\right) * \left(1 + \boldsymbol{R}_{fc}^{tmc}\right) - 1\right] \forall f \in (OSC, CA, CB), fc \in (AFS, CA, CB)$$

Note that the above equation indicates that the oldest share class returns $\widetilde{\boldsymbol{R}}_{f=OSC}^{tmc}$ are converted to the corresponding AFS currency $\boldsymbol{R}_{fc=AFS}^{tmc}$. This represents the point at which we map the OSC returns back to the corresponding AFS share class for which they proxy.

- Create a bivariate normal distribution described by the first two moments (mean and covariance matrix) of $\widetilde{\boldsymbol{R}}_f^{fc}$ and $\boldsymbol{R}_{fc}^{tmc}$. Sample from the bivariate distribution (such as 100,000 times) with replacement. Each draw will return two values: a sample return of the underlying AFS share class/category average/category benchmark, and a sample cross-currency return (AFS currency vs. target market currency). We combine these values to get a sampled underlying return in the target market currency:

$$\dot{\boldsymbol{R}}_f^{tmc} = \left[\left(1 + \dot{\boldsymbol{R}}_f^{fc}\right) * \left(1 + \dot{\boldsymbol{R}}_{fc}^{tmc}\right) - 1\right] \forall f, fc \in (AFS, CA, CB)$$

Where the $\dot{\boldsymbol{R}}$ nomenclature indicates samples from the bivariate distribution described above.

See Appendix B for details on the creation of the bivariate distribution.

- Calculate the CVaR by taking the mean of the worst *x*-percent of sampled outcomes. The *xth*-percentile below, which the mean is taken, is also known as the Variance-at-Risk (VaR).

$$\boldsymbol{CVaR}_f^{tmc} = \frac{1}{n_{r \le VaR}} * \sum_{j=0}^{\alpha} \dot{\boldsymbol{R}}_{f,r \le VaR}^{tmc} \ \forall f \ \in \ (AFS, CA, CB)$$

Where:

$\boldsymbol{CVaR}_f^{tmc}$ = a vector of CVaR estimates for each share class and category in $f \ \in \ (AFS, CA, CB)$;

$\dot{\boldsymbol{R}}_{f,r \le \alpha}^{tmc}$ = a matrix of all returns in f that are less than or equal to VaR;

$\mathrm{n}_{\mathrm{r \le VaR}}$ = the number of sampled returns less than or equal to the VaR[1];

- Average the AFS CVaR, Category Average CVaR, and Category Benchmark CVaR to get a final share class CVaR, $\overline{\boldsymbol{CVaR}}_{\mathrm{AFS}}^{\mathrm{tmc}}$:

$$\overline{\boldsymbol{CVaR}}_{AFS}^{tmc} = \frac{1}{n} * \sum \boldsymbol{CVaR}_f^{tmc} \ \forall f \ \in \ (AFS, CA, CB)$$

Where $n = 3$ if all three CVaRs exist.

Using the average CVaR serves two purposes. First, it "smooths" the CVaR calculation over time by incorporating more stable, category-level information. Second, in the event a CVaR cannot be calculated at the fund level (for reasons discussed further below), the share class will still get a CVaR derived from its associated category average and category benchmark returns.

3) Addressing Missing CVaRs

There are several reasons why an AFS share class, CA, or CB might not have a CVaR by this stage of the process:

- Insufficient Return History**:** For a return series to be evaluated using the LASSO/RBSA approach, it must have at least 36 months of history. When an oldest share class, category average, or category benchmark does not meet this requirement, a CVaR is not calculated for it.

- LASSO R-squared Threshold: LASSO regression results must yield an R-squared above a specified threshold (such as 70%) to be used to create a synthetic return history. A low R-squared indicates that

[1] By construction, each fund has the same number of monthly observations, so $n_{r \le \alpha}$ will be the same in each calculation.

none of the Global Indexes adequately describe the behavior of the return series being evaluated, and thus should not be relied upon. A CVaR is not calculated in these cases.

Note that as long as a CVaR can be calculated for at least one of the three return series (AFS, CA, CB), the AFS share class will receive a CVaR. In cases where none of the three return series is able to deliver a CVaR, the following process is applied:

- Category Peer Group (CPG) CVaR Sampling: Select the xth-percentile from the distribution of CVaRs already calculated, grouping by Morningstar Category.
- Asset Class Peer Group (ACPG) CVaR Sampling: Select the yth-percentile from the distribution of CVaRs already calculated, grouping by asset class.
- Take the average of the category and asset class peer group sampled CVaRs:

$$\overline{\boldsymbol{CVaR}}_{AFS}^{tmc} = \frac{1}{n} * \sum \boldsymbol{CVaR}_{f}^{tmc} \ \forall f \ \in \ (CPG, ACPG)$$

Where $n = 2$.

4) Assign Risk Rankings to Each Share Class in the AFS Universe Based on CVaR

- Order the vector $\overline{\boldsymbol{CVaR}}_{\mathrm{AFS}}^{\mathrm{tmc}}$ from least to most negative. Assign an integer value of 0 to 4 (called the "rank handle") to each AFS share class based on the percentile range in which that share class' CVaR falls within the ordered distribution of $\overline{\boldsymbol{CVaR}}_{\mathrm{AFS}}^{\mathrm{tmc}}$, based on the following mapping scheme:

Exhibit 2 Mapping of Percentile Ranges of the AFS CVaR Distribution to "Rank Handles"

CVaR Percentile Range	Rank Handle
0 - 10th percentile	0
10 - 30th percentile	1
30 - 55th percentile	2
55 - 80th percentile	3
80 - 100th percentile	4

Source: Morningstar

- For each AFS share class within a given percentile range, calculate the percentile rank of each share class CVaR (where percentile rank is between 0 and 1); add the percentile rank to the share class' rank handle to derive a final risk ranking for each share class out to two decimal places.

  For example, suppose share class *i*'s CVaR is at the 57th percentile of the distribution of all the AFS CVaRs. Using the mapping scheme described in Exhibit 2, we assign share class *i* a rank handle of 3 as its CVaR is within the 55-80th percentile range.

Within the 55-80th percentile range, share class *i*'s CVaR ranks at the 11th percentile. As such, we add 0.11 to the rank handle 3 to arrive at a share class risk ranking of 3.11. See Exhibit 3.

Exhibit 3 Stylized Example of AFS Share Class Risk Ranking Assignment

AFS Share Class		Percentile Rank All AFS CVaRs		Percentile Range		Rank Handle
f		54		30 - 55th		2
g		55		30 - 55th		2
h		56		56 - 80th		3
i	→	**57**	→	**56 - 80th**	→	**3**
j		58		56 - 80th		3
k		59		56 - 80th		3

↓

Percentile Rank Among Rank Handle = **3**		Final Risk Ranking
8		3.08
9		3.09
10		3.10
11	→	**3.11**
12		3.12
13		3.13

Source: Morningstar

5) Smoothing Model Risk Rankings

The steps outlined above result in a "raw" risk ranking based on returns from January 2003 through the most recent full month-end. Since each successive month uses all of the same underlying data (plus one extra month of returns), the majority of rankings tend to be fairly stable on a month-to-month basis. However, given the diversity of investment vehicles evaluated by the model, a subset of the AFS universe will typically exhibit pronounced month-to-month ranking volatility.

As a final step, we take the three-month moving average of the raw ranking to assign a final model risk ranking to each AFS share class.

Rules and Exceptions

Long-Short Funds

The constraint that all estimated weights in the LASSO/RBSA process must be positive is relaxed in cases where funds allow short positions. Because

Asset Class Restrictions

Once risk rankings have been assigned to each AFS share class, asset class constraints are imposed. As of May 2020, these include the following:

- Equity fund risk rankings must be 3 or greater;
- Fixed-income fund risk rankings (excluding Money Market) must be 2 or greater;

- Allocation fund risk rankings must be 2 or greater;
- Money market fund risk rankings cannot be greater than 2.

Analyst Input

The quantitative methodology is designed to capture the inherent risks of each fund, and we expect the methodology to be effective in most cases. However, we have incorporated several mechanisms to allow for analyst teams in the target market to influence and adjust the quantitative ranking process.

Category-Level Floor Rankings

Local analyst teams can provide minimum (floor) rankings based on asset class characteristics. Continuing the example above, assume Fund Share Class A is an emerging-markets equity fund. The local analyst team in the target market, being more familiar with the spirit of the regulatory environment, may deem ex ante that all emerging-markets equity funds should have a minimum risk ranking of 5. The quantitative model assigned this fund a ranking of 4, such that the local analyst floor rating becomes binding. The final risk ranking will thus be 5.

Exhibit 4 Stylized Example of a Binding Floor Ranking

Fund Share Class	CVaR	Risk Ranking (Unrounded)	Risk Ranking (Rounded)	Risk Ranking (with floor)
Fund Share Class A	-0.12	3.41	4	5

Source: Morningstar

Qualitative Overlay

In certain isolated cases, the emergence of newly developed asset class sectors means that there may be a lack of return history both at the fund and asset class level; one such example is the growing number of funds that invest in contingent convertible bonds. In such cases, analysts are empowered to apply a qualitative overlay to ensure the underlying risks of such funds are sufficiently captured in the risk ranking. It is important to note that analysts can apply only an upward adjustment from the original quantitative assessment.

Additional Operational Details

- Model Updates: Quantitative risk rankings are updated on a monthly basis along with new monthly return data for funds and indexes. Updating includes rerunning the LASSO-RBSA regression procedure, meaning regression results and index weightings can change over time, though month-to-month changes are likely to be minimal.

- Analyst Review: Risk ranking floor assignments will be reviewed on a semiannual basis, aligning with the existing semiannual Morningstar Category review process.

Appendix A: Global Index Selection

The list of global indexes used in the LASSO-RBSA process is intended to represent the investable universe, broadly defined. The list is curated according to the following guidelines:

- Asset class coverage: All major asset classes (for example, equities, fixed income) and subasset classes (for example, large-cap growth, high-yield corporate bonds) should be represented by at least one index.
- Region/country coverage: All major economies (for example, the U.S., eurozone, China) and regions (for example, Africa, Latin America) should be represented by at least one equity index and one fixed-income index, wherever possible.
- Sector coverage: Within equities, high-level global industry categories (for example, aerospace, pharmaceuticals) should be represented by at least one index.
- Currency hedges: For global equity and fixed-income indexes (for example, emerging-markets equities, global corporate bonds), include a version hedged with the following currencies wherever possible: U.S. dollar, euro, pound sterling, Japanese yen, Australian dollar, Korean won.

As of October 2020, we have identified over 300 indexes to broadly represent the investable universe. This list may change over time as new indexes become available.

Index Name	Asset Class	Provider
China Aggressive Allocation Benchmark	ALLOC	Morningstar
China Cautious Allocation Benchmark	ALLOC	Morningstar
Morningstar Balanced 20/80 GR USD	ALLOC	Morningstar
Morningstar Balanced 30/70 GR USD	ALLOC	Morningstar
Morningstar Balanced 35/65 GR USD	ALLOC	Morningstar
Morningstar Balanced 50/50 GR USD	ALLOC	Morningstar
Morningstar Balanced 65/35 GR USD	ALLOC	Morningstar
Morningstar Balanced 70/30 GR USD	ALLOC	Morningstar
Morningstar Balanced 80/20 GR USD	ALLOC	Morningstar
Morningstar CHN Alloc Fund TR CNY	ALLOC	Morningstar
Morningstar Gbl Allocation TR USD	ALLOC	Morningstar
CBOE Market Volatility (VIX)	EQUTY	Chicago Board Options Exchange
DJ South Asia TR USD	EQUTY	Dow Jones
FTSE China Yld	EQUTY	FTSE
FTSE Emerging Small Cap TR USD	EQUTY	FTSE
FTSE India Large Cap TR INR	EQUTY	FTSE
FTSE India Mid Cap TR INR	EQUTY	FTSE
Morningstar ASEAN ex-Vietnam GR USD	EQUTY	Morningstar
Morningstar Asia GR USD	EQUTY	Morningstar

Morningstar Asia Pac GR USD	EQUTY	Morningstar
Morningstar Asia xJpn GR USD	EQUTY	Morningstar
Morningstar Australia GR USD	EQUTY	Morningstar
Morningstar Brazil GR USD	EQUTY	Morningstar
Morningstar Canada GR CAD	EQUTY	Morningstar
Morningstar China GR CNY	EQUTY	Morningstar
Morningstar China Large Cap CNY	EQUTY	Morningstar
Morningstar CHN Equity Fund TR CNY	EQUTY	Morningstar
Morningstar CHN Middle Cap TR CNY	EQUTY	Morningstar
Morningstar CHN Small Cap TR CNY	EQUTY	Morningstar
Morningstar Dividend Leaders TR USD	EQUTY	Morningstar
Morningstar DM Asia Pac GR USD	EQUTY	Morningstar
Morningstar DM Asia Pac xJpn GR USD	EQUTY	Morningstar
Morningstar DM Bas Mat GR USD	EQUTY	Morningstar
Morningstar DM Com Svs GR USD	EQUTY	Morningstar
Morningstar DM Con Dfns GR USD	EQUTY	Morningstar
Morningstar DM Cons Cyc GR USD	EQUTY	Morningstar
Morningstar DM Energy GR USD	EQUTY	Morningstar
Morningstar DM Europe 100 GR EUR	EQUTY	Morningstar
Morningstar DM Europe Large Cap GR EUR	EQUTY	Morningstar
Morningstar DM Europe Large-Mid GR EUR	EQUTY	Morningstar
Morningstar DM Europe xUK GR EUR	EQUTY	Morningstar
Morningstar DM Fin Svs GR USD	EQUTY	Morningstar
Morningstar DM Healthcare GR USD	EQUTY	Morningstar
Morningstar DM Industrials GR USD	EQUTY	Morningstar
Morningstar DM Real Estate GR USD	EQUTY	Morningstar
Morningstar DM Technology GR USD	EQUTY	Morningstar
Morningstar DM Utilities GR USD	EQUTY	Morningstar
Morningstar DM xNA Tgt Mtum TR USD	EQUTY	Morningstar
Morningstar DM xNA Tgt Val TR USD	EQUTY	Morningstar
Morningstar EM Americas GR USD	EQUTY	Morningstar
Morningstar EM Asia GR USD	EQUTY	Morningstar
Morningstar EM Bas Mat GR USD	EQUTY	Morningstar
Morningstar EM Com Svs GR USD	EQUTY	Morningstar
Morningstar EM Con Dfns GR USD	EQUTY	Morningstar
Morningstar EM Cons Cyc GR USD	EQUTY	Morningstar
Morningstar EM Energy GR USD	EQUTY	Morningstar
Morningstar EM Europe GR EUR	EQUTY	Morningstar
Morningstar EM Fin Svc GR USD	EQUTY	Morningstar
Morningstar EM Ft Tilt GR USD	EQUTY	Morningstar
Morningstar EM GR USD	EQUTY	Morningstar
Morningstar EM Healthcare GR EUR	EQUTY	Morningstar

Morningstar EM Industrials GR USD	EQUTY	Morningstar
Morningstar EM Large Cap GR USD	EQUTY	Morningstar
Morningstar EM Real Estate GR USD	EQUTY	Morningstar
Morningstar EM Technology GR USD	EQUTY	Morningstar
Morningstar EM Utilities GR USD	EQUTY	Morningstar
Morningstar Eurozone 50 GR EUR	EQUTY	Morningstar
Morningstar Eurozone GR EUR	EQUTY	Morningstar
Morningstar Eurozone Large-Mid GR EUR	EQUTY	Morningstar
Morningstar Gbl Agricul Inputs GR USD	EQUTY	Morningstar
Morningstar Gbl Auto Mfg GR USD	EQUTY	Morningstar
Morningstar Gbl Banks-Diversified GR USD	EQUTY	Morningstar
Morningstar Gbl Bas Mat & En GR USD	EQUTY	Morningstar
Morningstar Gbl Biotechnology GR USD	EQUTY	Morningstar
Morningstar Gbl Chemicals GR USD	EQUTY	Morningstar
Morningstar Gbl Copper GR USD	EQUTY	Morningstar
Morningstar Gbl Drug Mfg -General GR USD	EQUTY	Morningstar
Morningstar Gbl Energy GR USD	EQUTY	Morningstar
Morningstar Gbl Gold GR USD	EQUTY	Morningstar
Morningstar Gbl Industrials GR USD	EQUTY	Morningstar
Morningstar Gbl Mkts GR USD	EQUTY	Morningstar
Morningstar Gbl Othr Indst Mtl&Mg GR USD	EQUTY	Morningstar
Morningstar Gbl Real Estate GR USD	EQUTY	Morningstar
Morningstar Gbl REIT - Diversd GR USD	EQUTY	Morningstar
Morningstar Gbl Semiconductors GR USD	EQUTY	Morningstar
Morningstar Gbl Steel GR USD	EQUTY	Morningstar
Morningstar Hong Kong GR USD	EQUTY	Morningstar
Morningstar India GR USD	EQUTY	Morningstar
Morningstar India Large-Mid GR USD	EQUTY	Morningstar
Morningstar Indonesia GR USD	EQUTY	Morningstar
Morningstar Japan GR JPY	EQUTY	Morningstar
Morningstar Japan Large Cap GR JPY	EQUTY	Morningstar
Morningstar Japan Large-Mid GR JPY	EQUTY	Morningstar
Morningstar Japan Small Cap GR JPY	EQUTY	Morningstar
Morningstar Korea GR KRW	EQUTY	Morningstar
Morningstar Mexico GR MXN	EQUTY	Morningstar
Morningstar Middle East & Africa GR USD	EQUTY	Morningstar
Morningstar MSCI Relative Value Small	EQUTY	Morningstar
Morningstar Nordic GR USD	EQUTY	Morningstar
Morningstar Russia GR USD	EQUTY	Morningstar
Morningstar Singapore GR USD	EQUTY	Morningstar
Morningstar South Africa GR USD	EQUTY	Morningstar
Morningstar Taiwan GR USD	EQUTY	Morningstar

Morningstar Turkey GR USD	EQUTY	Morningstar
Morningstar UK GR GBP	EQUTY	Morningstar
Morningstar UK Large Cap GR GBP	EQUTY	Morningstar
Morningstar UK Mid Cap GR GBP	EQUTY	Morningstar
Morningstar UK Small Cap GR GBP	EQUTY	Morningstar
Morningstar US Div Composite TR USD	EQUTY	Morningstar
Morningstar US Large Cap TR USD	EQUTY	Morningstar
Morningstar US Large Growth TR USD	EQUTY	Morningstar
Morningstar US Large Val TR USD	EQUTY	Morningstar
Morningstar US Mid Cap TR USD	EQUTY	Morningstar
Morningstar US Mid Growth TR USD	EQUTY	Morningstar
Morningstar US Mid Val TR USD	EQUTY	Morningstar
Morningstar US Small Cap TR USD	EQUTY	Morningstar
Morningstar US Small Growth TR USD	EQUTY	Morningstar
Morningstar US Small Val TR USD	EQUTY	Morningstar
Morningstar US Trgt Dividend TR USD	EQUTY	Morningstar
Morningstar US Trgt Momentum TR USD	EQUTY	Morningstar
Morningstar US Trgt Value TR USD	EQUTY	Morningstar
MSCI ACWI Volatility TILT GR USD	EQUTY	MSCI
MSCI EM Minimum Vol (USD) GR USD	EQUTY	MSCI
MSCI EM Small GR USD	EQUTY	MSCI
MSCI Europe High Div Yld GR EUR	EQUTY	MSCI
MSCI Frontier Markets GR USD	EQUTY	MSCI
MSCI India Growth GR USD	EQUTY	MSCI
MSCI India High Dividend Yield GR USD	EQUTY	MSCI
MSCI India Value GR USD	EQUTY	MSCI
MSCI World Minimum Vol (USD) GR USD	EQUTY	MSCI
S&P China Property TR USD	EQUTY	Standard & Poors
S&P EM Dividend Opportunities TR USD	EQUTY	Standard & Poors
S&P Europe LargeMid Growth TR USD	EQUTY	Standard & Poors
S&P Europe LargeMid Value TR USD	EQUTY	Standard & Poors
S&P Europe Small Growth TR USD	EQUTY	Standard & Poors
S&P Europe Small TR USD	EQUTY	Standard & Poors
S&P Global Dividend Opportunities TR USD	EQUTY	Standard & Poors
S&P Global Precs Mtls&Minls GR USD	EQUTY	Standard & Poors
S&P India Small TR USD	EQUTY	Standard & Poors
BBgBarc Asian Pacific 1-3 Yr TR JPY	FXINC	Bloomberg/Barclays
BBgBarc EM India Fixed Rate TR USD	FXINC	Bloomberg/Barclays
BBgBarc EM India Fl Rate TR USD	FXINC	Bloomberg/Barclays
BBgBarc EM India Intl Issue TR USD	FXINC	Bloomberg/Barclays
BBgBarc EM Intl Issue TR USD	FXINC	Bloomberg/Barclays
BBgBarc EM Local Issue TR USD	FXINC	Bloomberg/Barclays

BBgBarc EM USD Agg India MV TR USD	FXINC	Bloomberg/Barclays
BBgBarc EM USD Aggregate TR USD	FXINC	Bloomberg/Barclays
BBgBarc Euro Agg Mortgage TR EUR	FXINC	Bloomberg/Barclays
BBgBarc EuroDollar TR USD	FXINC	Bloomberg/Barclays
BBgBarc Gbl Agg Mortgages TR USD	FXINC	Bloomberg/Barclays
BBgBarc Gbl Agg Other Mortgages TR USD	FXINC	Bloomberg/Barclays
BBgBarc Global High Yield TR USD	FXINC	Bloomberg/Barclays
BBgBarc US Convertible Comp TR USD	FXINC	Bloomberg/Barclays
BBgBarc US Credit/Mortgage TR USD	FXINC	Bloomberg/Barclays
CSI Aggregate Bond 1-3 PR CNY	FXINC	China Securities Index Co.
CSI Aggregate Bond PR CNY	FXINC	China Securities Index Co.
CSI Financial Bond PR CNY	FXINC	China Securities Index Co.
CSI Treasury Bond PR CNY	FXINC	China Securities Index Co.
CSI Universal Bond PR CNY	FXINC	China Securities Index Co.
FTSE MTS IG Inflation-Linked TR EUR	FXINC	FTSE
FTSE MTS Inflation-Linked 10Y+ TR EUR	FXINC	FTSE
FTSE MTS Inflation-Linked 1-10Y TR EUR	FXINC	FTSE
FTSE MTS Inflation-Linked TR EUR	FXINC	FTSE
ICE BofA India Government TR USD	FXINC	ICE/Bank of America
Markit iBoxx ABF CN 10+ TR LCL	FXINC	Markit
Markit iBoxx ABF CN 1-3 TR LCL	FXINC	Markit
Markit iBoxx ABF CN 5-7 TR LCL	FXINC	Markit
Markit iBoxx ABF CN 7-10 TR LCL	FXINC	Markit
Markit iBoxx ABF CN TR LCL	FXINC	Markit
China Agg Bond Comp TR CNY	FXINC	Morningstar
China Conservative Comp CNY	FXINC	Morningstar
China Moderate Composite CNY	FXINC	Morningstar
Morningstar AU 1-5Y Core Bd GR AUD	FXINC	Morningstar
Morningstar CHN Fxd Inc Fund TR CNY	FXINC	Morningstar
Morningstar EM Corp Bd GR Hdg AUD	FXINC	Morningstar
Morningstar EM Corp Bd GR Hdg CAD	FXINC	Morningstar
Morningstar EM Corp Bd GR Hdg EUR	FXINC	Morningstar
Morningstar EM Corp Bd GR Hdg GBP	FXINC	Morningstar
Morningstar EM Corp Bd GR Hdg JPY	FXINC	Morningstar
Morningstar EM Corp Bd GR Hdg KRW	FXINC	Morningstar
Morningstar EM Corp Bd GR Hdg USD	FXINC	Morningstar
Morningstar EM Corp Bd GR USD	FXINC	Morningstar
Morningstar EM HY Corp Bd GR Hdg AUD	FXINC	Morningstar
Morningstar EM HY Corp Bd GR Hdg CAD	FXINC	Morningstar
Morningstar EM HY Corp Bd GR Hdg EUR	FXINC	Morningstar
Morningstar EM HY Corp Bd GR Hdg GBP	FXINC	Morningstar
Morningstar EM HY Corp Bd GR Hdg JPY	FXINC	Morningstar

Morningstar EM HY Corp Bd GR Hdg KRW	FXINC	Morningstar
Morningstar EM HY Corp Bd GR Hdg USD	FXINC	Morningstar
Morningstar EM HY Corp Bd GR USD	FXINC	Morningstar
Morningstar EM Sov Bd GR Hdg AUD	FXINC	Morningstar
Morningstar EM Sov Bd GR Hdg CAD	FXINC	Morningstar
Morningstar EM Sov Bd GR Hdg EUR	FXINC	Morningstar
Morningstar EM Sov Bd GR Hdg GBP	FXINC	Morningstar
Morningstar EM Sov Bd GR Hdg JPY	FXINC	Morningstar
Morningstar EM Sov Bd GR Hdg KRW	FXINC	Morningstar
Morningstar EM Sov Bd GR Hdg USD	FXINC	Morningstar
Morningstar EM Sov Bd GR USD	FXINC	Morningstar
Morningstar EZN 10+Y Core Bd GR USD	FXINC	Morningstar
Morningstar EZN 1-3Y Core Bd GR USD	FXINC	Morningstar
Morningstar EZN 5-10Y Core Bd GR USD	FXINC	Morningstar
Morningstar EZN Corp Bd GR USD	FXINC	Morningstar
Morningstar EZN Covered Bd GR Hdg EUR	FXINC	Morningstar
Morningstar EZN Covered Bd GR USD	FXINC	Morningstar
Morningstar EZN HY Bd GR EUR	FXINC	Morningstar
Morningstar EZN Trsy Bd GR USD	FXINC	Morningstar
Morningstar EZN Trsy Inf-Lnkd GR USD	FXINC	Morningstar
Morningstar Gbl Core Bd GR Hdg AUD	FXINC	Morningstar
Morningstar Gbl Core Bd GR Hdg CAD	FXINC	Morningstar
Morningstar Gbl Core Bd GR Hdg EUR	FXINC	Morningstar
Morningstar Gbl Core Bd GR Hdg GBP	FXINC	Morningstar
Morningstar Gbl Core Bd GR Hdg JPY	FXINC	Morningstar
Morningstar Gbl Core Bd GR Hdg KRW	FXINC	Morningstar
Morningstar Gbl Core Bd GR Hdg USD	FXINC	Morningstar
Morningstar Gbl Core Bd GR USD	FXINC	Morningstar
Morningstar Gbl Corp Bd GR Hdg AUD	FXINC	Morningstar
Morningstar Gbl Corp Bd GR Hdg CAD	FXINC	Morningstar
Morningstar Gbl Corp Bd GR Hdg EUR	FXINC	Morningstar
Morningstar Gbl Corp Bd GR Hdg GBP	FXINC	Morningstar
Morningstar Gbl Corp Bd GR Hdg JPY	FXINC	Morningstar
Morningstar Gbl Corp Bd GR Hdg KRW	FXINC	Morningstar
Morningstar Gbl Corp Bd GR Hdg USD	FXINC	Morningstar
Morningstar Gbl Corp Bd GR USD	FXINC	Morningstar
Morningstar Gbl Trsy Bd GR Hdg AUD	FXINC	Morningstar
Morningstar Gbl Trsy Bd GR Hdg CAD	FXINC	Morningstar
Morningstar Gbl Trsy Bd GR Hdg EUR	FXINC	Morningstar
Morningstar Gbl Trsy Bd GR Hdg GBP	FXINC	Morningstar
Morningstar Gbl Trsy Bd GR Hdg JPY	FXINC	Morningstar
Morningstar Gbl Trsy Bd GR Hdg KRW	FXINC	Morningstar

Morningstar Gbl Trsy Bd GR Hdg USD	FXINC	Morningstar
Morningstar Gbl Trsy Bd GR USD	FXINC	Morningstar
Morningstar Gbl Trsy Inf-Lnkd GR Hdg AUD	FXINC	Morningstar
Morningstar Gbl Trsy Inf-Lnkd GR Hdg CAD	FXINC	Morningstar
Morningstar Gbl Trsy Inf-Lnkd GR Hdg EUR	FXINC	Morningstar
Morningstar Gbl Trsy Inf-Lnkd GR Hdg GBP	FXINC	Morningstar
Morningstar Gbl Trsy Inf-Lnkd GR Hdg JPY	FXINC	Morningstar
Morningstar Gbl Trsy Inf-Lnkd GR Hdg KRW	FXINC	Morningstar
Morningstar Gbl Trsy Inf-Lnkd GR Hdg USD	FXINC	Morningstar
Morningstar Gbl Trsy Inf-Lnkd GR USD	FXINC	Morningstar
Morningstar Gbl xJpn Trsy Bd GR Hdg USD	FXINC	Morningstar
Morningstar Gbl xJpn Trsy Bd GR USD	FXINC	Morningstar
Morningstar Gbl xUS Trsy Bd GR Hdg USD	FXINC	Morningstar
Morningstar Gbl xUS Trsy Bd GR USD	FXINC	Morningstar
Morningstar Jpn 1-5Y Core Bd GR JPY	FXINC	Morningstar
Morningstar Jpn Corp Bd GR USD	FXINC	Morningstar
Morningstar Jpn Trsy Bd GR USD	FXINC	Morningstar
Morningstar Korea Trsy Bd GR USD	FXINC	Morningstar
Morningstar UK 1-3Y Core Bd GR GBP	FXINC	Morningstar
Morningstar UK Corp Bd GR USD	FXINC	Morningstar
Morningstar UK Gilt Bd GR USD	FXINC	Morningstar
Morningstar UK Trsy Inf-Lnkd GR USD	FXINC	Morningstar
Morningstar US 10+Y Corp Bd TR USD	FXINC	Morningstar
Morningstar US 10+Y TIPS TR USD	FXINC	Morningstar
Morningstar US 10+Y Tsy & Gv Bd TR USD	FXINC	Morningstar
Morningstar US 1-3Y Core Bd TR Hdg AUD	FXINC	Morningstar
Morningstar US 1-3Y Core Bd TR Hdg CAD	FXINC	Morningstar
Morningstar US 1-3Y Core Bd TR Hdg EUR	FXINC	Morningstar
Morningstar US 1-3Y Core Bd TR Hdg GBP	FXINC	Morningstar
Morningstar US 1-3Y Core Bd TR Hdg JPY	FXINC	Morningstar
Morningstar US 1-3Y Core Bd TR Hdg KRW	FXINC	Morningstar
Morningstar US 1-3Y Core Bd TR Hdg USD	FXINC	Morningstar
Morningstar US 1-3Y Core Bd TR USD	FXINC	Morningstar
Morningstar US 1-5Y Corp Bd TR USD	FXINC	Morningstar
Morningstar US 1-5Y TIPS TR USD	FXINC	Morningstar
Morningstar US 1-5Y Tsy&Gv Bd TR USD	FXINC	Morningstar
Morningstar US 5-10Y Corp Bd TR USD	FXINC	Morningstar
Morningstar US 5-10Y TIPS TR USD	FXINC	Morningstar
Morningstar US 5-10Y Tsy&Gv Bd TR USD	FXINC	Morningstar
Morningstar US ABS TR USD	FXINC	Morningstar
Morningstar US HY Bd TR Hdg AUD	FXINC	Morningstar
Morningstar US HY Bd TR Hdg CAD	FXINC	Morningstar

Morningstar US HY Bd TR Hdg EUR	FXINC	Morningstar
Morningstar US HY Bd TR Hdg GBP	FXINC	Morningstar
Morningstar US HY Bd TR Hdg JPY	FXINC	Morningstar
Morningstar US HY Bd TR Hdg KRW	FXINC	Morningstar
Morningstar US HY Bd TR Hdg USD	FXINC	Morningstar
Morningstar US HY Bd TR USD	FXINC	Morningstar
Morningstar US MBS TR USD	FXINC	Morningstar
RMB 12 month Lump-Sum Deposit Rate	FXINC	People's Bank of China
SSE Corporate Bond PR CNY	FXINC	Shanghai Stock Exchange
SSE Treasury Bond PR CNY	FXINC	Shanghai Stock Exchange
S&P US HY Low Volatility Corp Bd TR USD	FXINC	Standard & Poors
Credit Suisse Managed Futures USD	MISCL	Credit Suisse
Morningstar Agriculture Cmdty TR USD	MISCL	Morningstar
Morningstar Broad Hedge Fund TR USD	MISCL	Morningstar
Morningstar CHN OE Fund TR CNY	MISCL	Morningstar
Morningstar Cmdty Curr TR USD	MISCL	Morningstar
Morningstar EM Real Estate PR USD	MISCL	Morningstar
Morningstar Gbl Lng/Shrt Cmdty TR USD	MISCL	Morningstar
Morningstar Gbl Lng/Shrt Curr TR USD	MISCL	Morningstar
Morningstar Gbl Lng-Only Cmdty TR USD	MISCL	Morningstar
Morningstar MSCI Long-Short Crdt Offshre	MISCL	Morningstar
SSE Misc Sub PR CNY	MISCL	Shanghai Stock Exchange
SSE Property PR CNY	MISCL	Shanghai Stock Exchange
S&P GSCI Precious Metal TR	MISCL	Standard & Poors
SIBOR 3 Month SGD	MNMKT	Association of Banks of Singapore
Chile PDBC 30 Day Monthly	MNMKT	Banco Central de Chile
Brazil CDI Yld BRL	MNMKT	Banco Central do Brasil
Taiwan Bank 3 Month Deposit Rate	MNMKT	Bank of Taiwan
USTREAS T-Bill Auction Ave 3 Mon	MNMKT	Federal Reserve
FBIL MIBOR Overnight INR	MNMKT	FIMMD of India
ICE BofA AUD Overnight Offer TR USD	MNMKT	ICE/Bank of America
ICE BofA CAD Overnight Offer TR USD	MNMKT	ICE/Bank of America
ICE BofA CHF 1M Dep OR CM TR LOC	MNMKT	ICE/Bank of America
ICE BofA DKK 1M Dep OR CM TR LOC	MNMKT	ICE/Bank of America
ICE BofA HKD 1M Dep OR CM TR LOC	MNMKT	ICE/Bank of America
ICE BofA MYR 1M Dep OR CM TR LOC	MNMKT	ICE/Bank of America
ICE BofA NOK 1M Dep OR CM TR LOC	MNMKT	ICE/Bank of America
ICE BofA SARON Overnight Offer TR USD	MNMKT	ICE/Bank of America
ICE BofA SEK 1M Dep OR CM TR LOC	MNMKT	ICE/Bank of America
ICE BofA SOFR Overnight Offer TR USD	MNMKT	ICE/Bank of America
ICE BofA SONIA Overnight Offer TR USD	MNMKT	ICE/Bank of America
ICE BofA TONAR Overnight Offer TR USD	MNMKT	ICE/Bank of America

KBP CD KRW	MNMKT	Korea Asset Pricing
Morningstar AU Cash GR USD	MNMKT	Morningstar
Morningstar Can Cash GR USD	MNMKT	Morningstar
Morningstar EZN Cash GR USD	MNMKT	Morningstar
Morningstar Jpn Cash GR USD	MNMKT	Morningstar
Morningstar UK Cash GR USD	MNMKT	Morningstar
Morningstar US Cash TR USD	MNMKT	Morningstar
RMB 3 month Lump-Sum Deposit Rate	MNMKT	People's Bank of China
Israel T-Bill 3 Month	MNMKT	Primary Rate Index
NZ 90 Day Treasury Bills Issue Rate NZD	MNMKT	Reserve Bank of New Zealand
JIBAR 1 Month ZAR	MNMKT	South African Reserve Bank
ThaiBMA 91 day T-bill TR THB	MNMKT	Thai Bond Market Association

Appendix B: Calculation Details

Least Absolute Shrinkage and Selection Operator

LASSO regression is a subset of a broader family of regularized regression techniques. Ordinary least squares, or OLS, regressions are estimated by minimizing the squared error between the observed values of the dependent variable y and the fitted values of y:

$$\min_{\beta}\left\{\sum_{i=1}^{n}\left(y_i-\sum_{j=1}^{p}x_{ij}\beta_j)\right)^2\right\}$$

Where:

y_i = The i-th observation of the dependent variable y
x_{ij} = The i-th observation of the independent variable x_j
β_j = The estimated coefficient of independent variable x_j
n = The number of observations
p = The number of explanatory variables/estimated coefficients

A LASSO regression is estimated by minimizing the squared error between the observed values $\mathbf{y}$ and the fitted values of $\mathbf{y}$, augmented with a constraint that seeks to minimize the sum of the absolute values of the coefficients:

$$\min_{\beta}\left\{\sum_{i=1}^{n}(y_i-\sum_{j=1}^{p}x_{ij}\beta_j)^2\right\}\ subject\ to\ \sum_{j=1}^{p}|\beta_j|\leq t$$

In this case, t is the maximum sum of the absolute values of the estimated coefficients β_j. The LASSO equation can also be expressed in Lagrangian form with multiplier λ:

$$\min_{\beta}\left\{\underbrace{\sum_{i=1}^{n}\left(y_i-\sum_{j=1}^{p}x_{ij}\beta_j\right)^2}_{OLS}+\underbrace{\lambda\sum_{j=1}^{p}\left|\beta_j\right|}_{LASSO\ penalty}\right\}$$

Where the second term in brackets acts is called the "penalty" term. Given the constraint/penalty, the coefficients β_j in the LASSO are estimated using a "soft-thresholding" function $S(\hat{\beta},\lambda)$ of the form:

$$\hat{\beta}^{LASSO}(\lambda)=S(\hat{\beta},\lambda)=\begin{cases}\hat{\beta}^{OLS}-\lambda, if\ \hat{\beta}^{OLS}>0\ and\ \left|\hat{\beta}^{OLS}\right|>\lambda,\\ \hat{\beta}^{OLS}+\lambda, if\ \hat{\beta}^{OLS}<0\ and\ \left|\hat{\beta}^{OLS}\right|>\lambda,\\ 0, if\ \left|\hat{\beta}^{OLS}\right|\leq\lambda.\end{cases}$$

Where:

$\hat{\beta}^{OLS}$ = the estimated coefficient from a generic OLS process.

Thus, every OLS-estimated beta coefficient is reduced in absolute terms by the penalization parameter λ. Any OLS beta that is smaller (in absolute terms) than λ is reduced to zero.

The appropriate penalization parameter λ is determined using an iterative method called coordinate descent: A relatively large penalization parameter λ is selected and, using the soft-thresholding function, LASSO coefficients are estimated. As is clear from the soft-thresholding logic, the larger the penalization parameter λ, the more estimated coefficients will be reduced to zero. The process repeats, using successively smaller penalization parameters λ, until the sum of squared errors cannot be reduced further, that is, until the errors are minimized. For more information on techniques to solve LASSO and similar problems, see Friedman et al. (2007).[2]

Bivariate Normal Distribution

The probability density function (PDF) of a bivariate normal distribution takes the form:

$$P(x_1,x_2)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}*e^{\left[\frac{-z}{2(1-\rho^2)}\right]}$$

Where:

$$z=\frac{(x_1-\mu_1)^2}{\sigma_1^2}-\frac{2\rho(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2}+\frac{(x_2-\mu_2)^2}{\sigma_2^2}$$

$$\rho=\frac{Cov_{12}}{\sigma_1\sigma_2}$$

We use the empirical returns (or the Lasso-RBSA-derived synthetic returns) of each share class, category average, and category benchmark in its base currency to define the mean μ_1 and standard deviation σ_1

[2] Friedman, J., Hastie, T., Hofling, H., & Tibshirani, R. 2007, "Pathwise Coordinate Optimization." Ann. Applied Statistics, Vol. 1, No. 2, P. 302. https://arxiv.org/pdf/0708.1485.pdf

and the empirical return of the target currency relative to the base currency to define the mean μ_2 and standard deviation σ_2. These two return series are used to calculate the covariance matrix Cov_{12}.

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For More Information

+1 312 244-7541
lee.davidson@morningstar.com

MORNINGSTAR®

22 West Washington Street
Chicago, IL 60602 USA