Market-Driven Scenario Analysis
Methodology

Overview
The Market-Driven Scenario Analysis tool allows users to select a market index to determine the impact of user-specified market shocks on factor exposures, portfolio returns, Value at Risk, or VaR, and Conditional Value at Risk, or CVaR.

As of July 2018, the S&P 500 is down by nearly 5% for the year and the jury is still out on the impact of the trade war. What if the S&P 500 drops another 10% or 20%? How will that affect a portfolio?

Exhibit 1 Forecasts Returns for American Funds Growth Fund of America Where S&P 500 Declines By 20%

Exhibit 1 plots the forecast returns as well as the volatility to answer just such a question. Here, we consider how much American Funds Growth Fund of America Class A will lose if the S&P 500 loses 25% of its value and how long it will take. In this scenario, we are looking at the first quartile, or 25th percentile, of S&P 500 returns to determine the time horizon for a 25% crash in the index. If we consider the lowest 10th percentile, as shown in Exhibit 2, the time horizon for the 25% crash is shortened and...

the volatility decreases. With the Market-Driven Scenario Analysis tool, the user can select the quantile dividing the distribution of returns of any selected index and, in doing so, specify the market conditions for the shock.

The user can also specify the number of weeks for the scenario to occur. For example, let’s say we want to consider what will happen if the S&P 500 falls by 10% in 12 weeks. The Market-Driven Scenario Analysis tool will determine the percentile under which such a scenario can occur and calculate the impact on a portfolio.

For stress-testing purposes, the user can specify the number of weeks and market-stress conditions to calculate the impact on an investment option. So, instead of specifying how much an index will fall, we can simply look at what will happen to a portfolio for four weeks under market-stress conditions, which is represented by the lower quartile of the distribution, for example.

In summary, the user can select two of the following three parameters to specify the market-stress conditions: percentile of the distribution, market shock, and shock duration. The mean return and volatility of a given portfolio are then calculated along with a mean alpha, the 95% confidence interval associated with the mean, as well as the VaR and the CVaR. Given that the market shock can be either positive or negative, the VaR and CVaR are calculated only for negative shocks.

Exhibit 2 Forecast Returns for American Funds Growth Fund of America Where S&P 500 Declines By 20% Using the Lowest 10th Percentile

Market Indexes
The user can select a market index to include in the model and the start date of the analysis. The index data is first normalized by the lagged values of the index. If successive values are identical, the normalized value of 1 is dropped from the subsequent analysis.

Market Shock
The returns are resampled on a weekly basis, and the cumulative weekly product is calculated. The rank \( r \) of the user-defined percentile \( p \) is calculated by \( r = p(N - 1) + 1 \) where \( N \) is the total number of the returns in the index. The threshold is simply the value of the return in an ordered list plus the fractional part of the rank multiplied by the difference between the values of the next rank. For example, if the returns consist of the ordered list 1.01, 1.02, 1.03, and 1.04, and the user selects the 10th percentile, the rank \( r = 0.1(4-1) + 1 = 1.3 \) and the threshold is equal to \( 1.0v_1 + 0.3(v_2 - v_1) = 1.013 \) where \( v_1 = 1.01 \) and \( v_2 = 1.02 \).

Exhibit 3 shows the normalized weekly returns of the S&P 500 beginning in January 2003; the red bars indicate the index returns below the 25th percentile.

Exhibit 3 Histogram of the Weekly Returns of Normalized S&P 500 Returns With Percentile Cut

If the user-defined percentile is less than or equal to 0.5, the lower 50th percentile of the index distribution is used for determining the mean returns and the user-defined market shock represents a negative percentage of market movement. Conversely, if the percentile is greater than 0.5, the upper 50th percentile of the index distribution is used for determining the mean returns and the market shock input represents the percentage of positive returns.
If the user specifies the duration \( n \) and magnitude \( x \) of the shock, the percentile of the distribution is calculated according to a binary search algorithm where the mean \( \mu \) of the stressed index is

\[
\mu = 10^{\log x/n}.
\]

**Volatility Clustering**

Because markets in freefall often have high volatility, the distribution should include both the significant rises and falls. To capture this volatility clustering, the weekly returns below the percentile threshold are selected and the daily returns from that week are retrieved, as shown in Exhibit 4. The risk factor premiums on those dates are also collected.

**Exhibit 4** Histogram of Daily Returns From the Normalized Weekly Returns for the S&P 500

![Histogram of Daily Returns From the Normalized Weekly Returns for the S&P 500](image)

*Source: Morningstar, Inc. Data as of Feb. 6, 2018.*

**Sample Size for Estimating a Mean**

The logarithm of any arbitrary base \( k \) can be computed through the formula,

\[
\log_b x = \frac{\log_k x}{\log_k b} = \frac{\log_{10} x}{\log_{10} b}\]

With the mean \( \mu \) of the daily scenario returns retrieved from the user-defined quantile of the normalized index returns, the sample size \( n \) necessary for the user-specified market shock \( x \) can be obtained by
Because the weekly returns are used to capture volatility clustering, the sample size can be interpreted as the number of weeks required to obtain the user-specified market shock \( x \). If the shock duration is specified by the user, the sample size is the shock duration \( n \).

**Bootstrap Risk Factor Premiums**

The sampling distributions of scenario premiums below (or above if the shock is positive) the user-defined percentile are resampled with replacement 10,000 times through the Monte Carlo method to estimate the mean and volatility of the shocked risk factor premiums.

**Exhibit 5** Forecast Returns for American Funds Growth Fund of America Where the S&P 500 Declines By 20%

![Chart showing forecast returns for American Funds Growth Fund of America](chart.png)


**Total Return Forecast and Alpha**

With the shocked risk factor premiums, the total return of a given portfolio is calculated based on the last available factor exposures:

\[
    r_{t+n} = X_t f_{t+n}. 
\]

where \( r_{t+n} \) is the return of a portfolio, \( t \) is the most recent day, \( n \) represents the duration of the scenario analysis forecast, \( X_t \) is a \( 1 \times p \) vector of factor exposures at time \( t \), \( p \) is the number of factors, and \( f_{t+n} \) is a vector of forecast risk factor premiums at time \( t + n \). Since there is a distribution of
10,000 returns for each premium, there is also a distribution of 10,000 returns at the portfolio level based on factor exposures at time $t$ as shown in Exhibit 5. The benchmark returns are calculated in the same way.

With the mean portfolio and benchmark returns, alpha is calculated along with a 95% confidence interval.

**Value at Risk and Conditional Value at Risk**

VaR is a measure of the potential of loss with a given probability. Based on the outcome of the market-driven shock, the VaR is calculated as the value of the portfolio return at the user-specified confidence interval. If the user selects a 95% confidence level for VaR, the portfolio return at the 5th percentile is calculated. CVaR is another risk measure that represents the mean loss of a portfolio occurring below the user-specified percentile. Therefore, given a 95% confidence level, the CVaR is the mean of the bootstrapped portfolio returns below the 5th percentile, as indicated in Exhibit 6.

**Exhibit 6** Histogram of Returns for American Funds Growth Fund of America Where the S&P 500 Declines by 20%

![Histogram of Returns](source: Morningstar, Inc. Data as of Feb. 6, 2018.)

VaR = -0.38
CVaR = -0.42
**Time to Recovery**

If the mean of the portfolio returns for a user-defined scenario are negative, a recovery scenario is run to estimate the time for the portfolio to recover its value. The new scenario market movement percentage is defined as $1/(1+\text{mean returns})$. For example, if the mean of the portfolio returns for a user-defined market shock is negative 0.2, the recovery scenario market movement is $1/0.8 = 1.25$, or a 25% increase in returns. The distribution for the recovery is the cumulative weekly product of the index returns above the 10th percentile. The time to recovery is based on the how long it will take the mean of the portfolio to return to its original value.
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