Estimates of Small Stock Betas are Much Too Low

Adjusted estimates of beta are positively related to future common stock returns.

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Abstract

The authors adjust estimates of systematic risk, betas, for cross-autocorrelations in security returns. They show that substantial positive adjustments to beta are necessary for small firms. Traditional estimates of beta are unrelated to future returns over the 1931 through 1994 time period, whereas adjusted estimates are positively correlated with future returns. In addition, adjusted beta estimates partially account for the size effect in common stock returns.
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Financial economists have long been aware that nonsynchronous prices and other market frictions can cause positive autocorrelation in portfolio returns. Lo and MacKinlay [1990a] document that returns to portfolios of small company stocks (measured by market value of outstanding equity) tend to lag returns to portfolios of large company stocks. On the other hand, lagged returns on small company stocks are not correlated with contemporaneous returns on large company stocks. Moreover, McQueen, Pinegar, and Thorley [1996] and Peterson and Sanger [1995] show that these “cross-autocorrelations” exist even when returns are sampled at monthly intervals. The objective of this article is to illustrate the importance of accounting for autocorrelation in returns when estimating systematic risk.

The capital asset pricing model (CAPM) relates the expected return on a security, or a portfolio, to its systematic risk, beta. Estimates of beta represent a critical step in the process of estimating the cost of equity capital for a firm. Estimates of the cost of equity are typically used as inputs in the capital budgeting process of corporations and in business valuations. Beta estimates are also used by portfolio managers. Estimates of beta can be used to form portfolios which differ in terms of risk and expected return. In addition, accurate estimates of beta are essential for the proper evaluation of managerial performance. Currently, no commercial beta service provides beta estimates that account for cross-autocorrelations in security returns.¹

The beta coefficient for a security is typically estimated as the slope coefficient from a regression of historical security returns, or returns in excess of the risk-free rate, on contemporaneous market returns. We adjust the traditional estimate of beta to account for cross-autocorrelations in returns by incorporating the information contained in past market returns. That is, we estimate beta as the sum of the slope coefficients from a regression of excess security returns on contemporaneous and lagged excess market returns. Therefore, the adjusted beta methodology is easily implemented.
We show that substantial positive adjustments to beta are necessary for small firms. Therefore, we conclude that traditional estimates of beta are severely biased downward for these firms. We then show that traditional estimates of beta are uncorrelated with future stock returns over the 1931 through 1994 time period. On the other hand, adjusted beta estimates are positively correlated with future security returns. Moreover, we show that adjusted estimates of beta partially explain the size effect in common stock returns. Based on our results, we recommend that providers of beta estimates incorporate the information contained in lagged market returns into their estimates of systematic risk.

Firm Size and Beta Estimation

Our first task is to document the impact of portfolio autocorrelations on monthly estimates of beta. Based on recent research, we expect that the magnitude of the autocorrelation in returns will vary predictably with firm size. Specifically, the returns on small firm portfolios should exhibit a greater degree of autocorrelation than the returns on large firm portfolios.2

Why does the degree of autocorrelation vary inversely with firm size? Size most likely proxies for other factors which, in turn, cause the observed return autocorrelations. For example, size is related to the bid-ask spread. The speed of adjustment to new information is likely to be negatively related to the bid-ask spread. Prices of high spread stocks respond more slowly to new information because it is not profitable for traders to act upon new information until the value of that information exceeds the costs of transacting.3 Therefore, the returns of high spread firms are more likely to exhibit positive autocorrelation.

Autocorrelation in returns can also result from nonsynchronous trading. Nonsynchronous trading refers to the fact that not all stocks trade at regular intervals. Beta estimation requires calculating a holding period return. The primary input into the calculation of return is closing price. However, an observed closing price may be associated with a trade that occurred much earlier in the period, or in a prior period. Therefore, the period over which firm
specific returns are measured may not be exactly aligned with the period over which the market return is measured. This problem tends to be more severe for small firms. Nonsynchronous trading, however, is not likely to be a severe problem when returns are sampled monthly. In addition, researchers have found that the autocorrelations in security returns cannot be completely explained by nonsynchronous trading.4

Finally, size could be related to the number of analysts that follow a particular firm. The speed of price adjustment to private information should increase with the number of firm analysts.5 The prices of firms with many analysts quickly respond to private information, whereas the prices of firms with few analysts respond more slowly to new information. Therefore, the returns of few analyst firms are more likely to exhibit positive autocorrelation.

To illustrate the role that autocorrelations and cross-autocorrelations play in beta estimation, we estimate the traditional CAPM beta and an adjusted beta for ten size-based portfolios of NYSE/AMEX/NASDAQ stocks over the 1926 through 1994 time period. AMEX and NASDAQ stocks are incorporated into the portfolios beginning in 1962 and 1972 respectively. We focus on firm size because complete information on the number of analysts and bid-ask spread is not readily available for all firms. The data used throughout this article are from Ibbotson Associates.

Size portfolio returns are constructed by the Center for Research in Security Prices (CRSP). CRSP excludes unit investment trusts, closed-end funds, real estate investment trusts, americus trusts, foreign stocks, and American Depository Receipts from the portfolios. Moreover, CRSP uses NYSE firms only to determine the size breakpoints for the portfolios. Specifically, all eligible NYSE stocks are ranked by firm size (market value of outstanding equity) and then split into ten equally populated groups, or deciles. The largest companies are put into decile 1 and the smallest are put into decile 10. The capitalization for the largest company in each decile serves as the breakpoint for that decile. Breakpoints are re-balanced on the last day of trading in
March, June, September, and December of each year. NYSE/AMEX/NASDAQ firms are then assigned to the portfolios using the decile breakpoints. Monthly portfolio returns are market capitalization weighted averages of the individual returns within each of the ten portfolios.

Two estimates of systematic risk are calculated in this study. The traditional beta, $\beta$, is the slope coefficient from the regression

$$ R_t - R_{ft} = \alpha + \beta(R_{mt} - R_{ft}) + \epsilon_t, $$

where $R_t$ is the return on a portfolio, or security, over time $t$, $R_{mt}$ is the contemporaneous market return, and $R_{ft}$ is the risk-free rate at time $t$. We use the CRSP market capitalization weighted index of NYSE/AMEX/NASDAQ firms as our estimate of market returns. The 30-day Treasury bill, observed at the beginning of the month, is used as the estimate of the risk-free rate.

We adjust the estimate of beta for cross-autocorrelations by first regressing an asset's excess returns on both current and lagged excess market returns. That is, we run the regression

$$ R_t - R_{ft} = \alpha + \beta(R_{mt} - R_{ft}) + \beta_{-1}(R_{mt-1} - R_{ft-1}) + \epsilon_t. $$

The adjusted estimate of systematic risk, $SUM\beta$, is then defined as the sum of the slope coefficients from the multiple regression (2). Therefore, $SUM\beta = \beta + \beta_{-1}$.

To understand the choice of incorporating a lagged market return, recall that prices of large, high information, low transaction cost firms adjust quickly to new information. So the returns on these firms exhibit little autocorrelation. On the other hand, the prices of small, low information, high transaction cost firms adjust slowly to new information. Therefore, the returns on these firms may exhibit positive autocorrelation. Using lagged returns on a market index that is heavily weighted toward large, high information, liquid firms will capture the delayed adjustment of small, low information, high transaction cost firms to market wide information. So $\beta$ measures the contemporaneous covariation, i.e., comovement, between the returns on a security and the market portfolio. The component of a security’s current period return that
reflects market movements from the prior period is captured by the coefficient $\beta_{-1}$. Since covariances are additive, $\sum \beta = \beta + \beta_{-1}$ captures the total covariation between the returns on a security and current and lagged market returns. In general, $\beta_{-1}$ increases as firm size decreases. So the relative weights given to contemporaneous market returns and lagged market returns in the $\sum \beta$ calculation depend upon firm size. An important advantage of the adjusted estimate of systematic risk $\sum \beta$ is that it is easy to calculate.\(^6\)

Table 1 reports both estimates of systematic risk and average portfolio excess returns during the 1926 through 1994 sample period for the 10 size portfolios. We expect the risk and return of small firms to exceed the risk and return of large firms. To understand this conjecture, consider an economy where investors tradeoff risk and return. Suppose there are two firms, A and B, that are identical in terms of expected future cash flows. However, the cash flows of firm A are more risky, in terms of systematic risk, than the cash flows of firm B. Firm A will have a lower price, higher expected rate of return and will also be smaller in terms of market value of outstanding equity than firm B. Therefore, when size is measured as market value of outstanding equity, small firms, in general, will be more risky and have higher returns than large firms.\(^7\)

From Table 1, excess returns more than double from the largest portfolio (portfolio 1) to the smallest (portfolio 10). To be consistent with the CAPM, beta should increase proportionally with excess returns. That is, beta should also double. The traditional estimate $\beta$ gradually increases from a low of 0.93 for portfolio 1 to a high of 1.57 for portfolio 10. Thus, the traditional estimate of beta does not increase proportionally with the excess return. On the other hand, $\sum \beta$ consistently increases from 0.91 for the largest portfolio to 1.84 for the smallest. The difference between $\sum \beta$ and $\beta$ increases from -0.02 for the largest portfolio to 0.27 for the smallest. The coefficient on the lagged market return is significantly greater than zero for
portfolios 4 through 10. Figure 1 plots $\beta$ and $\text{SUM}\beta$ for each size portfolio. The difference between $\beta$ and $\text{SUM}\beta$ clearly widens as firms get progressively smaller.

The practical significance of the results in Table 1 can be illustrated with a simple example. Suppose we wanted to estimate the cost of equity capital for a firm in portfolio 10. Setting the market risk premium at its long-run average value of approximately 7 percent and using the CAPM to estimate the cost of equity implies that the cost of equity estimate is about 2 percent higher when the adjusted beta is used in lieu of the traditional estimate of beta. Therefore, these results suggest that traditional estimates of beta are severely understated for small firms.

**Beta as a Predictor of Returns**

If $\text{SUM}\beta$ contains less estimation error than $\beta$, then $\text{SUM}\beta$ should be a better predictor of future returns. The primary implication of the CAPM is that the expected excess return on a security is directly related to its beta. The standard approach to testing this hypothesis is the Fama and MacBeth [1973] cross-sectional regression approach. In the Fama-MacBeth approach, the cross-section of realized returns for a given month is regressed against beta estimates using the following regression equation:

$$R_t - R_{ft} = \lambda_{t0} + \lambda_{t1}b_t + z_t,$$

(3)

where $\lambda_{t0}$ and $\lambda_{t1}$ are the regression intercept and slope respectively for month $t$ and $b_t$ is the time $t$ estimate of beta for the asset. The regression slope coefficient for a given month $t$ is interpreted as the risk premium per unit of beta risk for that month. The time-series mean of these regression slopes is used to test the hypothesis that, on average, beta risk is priced.

An important point to keep in mind is that the CAPM says that beta is related to expected return, not the actual, or realized, return. Actual returns contain an unexpected component which
can cause these returns to substantially deviate from expected returns. Therefore, researchers typically test the risk-return relationship over long time periods.

A second point is that estimates of beta are used in tests of the beta-return relationship. Therefore, obtaining accurate estimates of beta represents a critical step in assessing the risk-return tradeoff. Since betas estimated from portfolios are more precise, most researchers test the beta-return relationship using portfolios rather than individual securities. Moreover, betas estimated over longer time-horizons are more precise. Therefore, in recent research portfolio betas are estimated once over the entire sample period. Then these portfolio betas are used in each cross-sectional regression. In this study, we improve beta estimates by incorporating the information contained in past market returns. We begin our analysis by estimating the cross-sectional regressions using firm specific data. In addition, we estimate a $\beta$ and a $\text{SUM}\beta$ for each firm using historical data. For comparison purposes, we then estimate the cross-sectional regressions using full-period portfolio betas and portfolio returns. Each method will be explained in detail below.

Beginning in January of 1931, we calculate a $\beta$ and a $\text{SUM}\beta$ for all NYSE/AMEX/NASDAQ ordinary common stocks using the past 60 months of historical return data. Beta estimates are obtained via regressions (1) and (2) and are updated once per year. Using monthly data to estimate beta is a standard in practice as well as in academic research. We then estimate (3) using $\beta$ as the measure of risk. The above process is repeated each month through 1994 (768 months). The time-series mean of the monthly regression slopes is then used to test the hypothesis that beta is positively correlated with future returns. The cross-sectional regressions are repeated using $\text{SUM}\beta$ as the measure of risk. The results are reported in Panel A of Table 2.
The traditional estimate of beta shows no ability to explain returns in our test. The average slope coefficient on \( \beta \) implies an annual risk premium of more than 3 percent per year for one unit of \( \beta \) risk \((0.26 \times 12)\). However, the standard error on this coefficient is high so that we cannot say with any statistical confidence that the premium for risk is greater than zero. The regression coefficient on \( \text{SUM}\beta \) is statistically greater than zero at conventional levels. Note that the premium for one unit of \( \beta \) or \( \text{SUM}\beta \) risk is approximately the same. However, the coefficient on \( \text{SUM}\beta \) is more precise in the sense that it has a smaller standard error. Given the increased precision of this coefficient, we can reject the statistical hypothesis of no beta-return tradeoff in favor of a positive beta-return relationship in common stock returns.

We also examine the risk-return tradeoff using portfolios instead of individual securities. For each year, beginning in 1931, we first classify firms into portfolios based on their historical \( \beta \). The 5 percent of the firms with the smallest \( \beta \)s are grouped into portfolio 1 and the 5 percent with the largest \( \beta \)s are grouped into portfolio 20. An equal-weighted, “post-ranking” return for each month of the following year is then calculated for each portfolio.\(^9\) New estimates of beta are calculated each year so that the composition of the portfolios can change year by year. Fama-MacBeth point out that this beta sorting procedure yields portfolio betas that are biased since the highest and lowest portfolio betas are likely to be those with the most estimation error. Therefore, portfolio betas are re-estimated using the post-ranking portfolio returns. Moreover, it is now common practice to estimate the 20 portfolio betas over the entire sample period and use these estimates of beta in each cross-sectional regression.\(^10\) The process is repeated using \( \text{SUM}\beta \). The results are reported in Panel B of Table 2.

The results in Panel B support our earlier finding. That is, \( \beta \) is not significantly related to future returns, whereas \( \text{SUM}\beta \) is positively related to returns. The primary difference between the results reported in Panel A and those reported in Panel B is that the premium for
beta risk appears higher when portfolio betas are used for the risk-return tests. The slope coefficients on beta imply an annual risk premium per unit of beta risk of about 5 percent (0.39 × 12). The main point of the results reported in Table 2, however, is that we find a positive tradeoff between beta risk and return when the information incorporated in lagged market returns is incorporated into the beta estimate.

In a recent paper, Chan and Lakonishok [1993] find that the traditional estimate of beta is significantly related to returns over the 1932 through 1991 time period. They also show that the relationship between beta and returns is very sensitive to the sample period. Our results differ from Chan and Lakonishok’s for three reasons. First, our sample period is 1931 through 1994. Second, we include NASDAQ firms in our tests. We incorporate NASDAQ firms into our tests because these firms are an important part of a portfolio managers investment opportunity set. Since NASDAQ firms tend to be relatively small, incorporating a lag is extremely important for these firms. Finally, Chan and Lakonishok use a slightly different methodology for estimating beta. Specifically, they estimate beta over an initial 36 month period and then re-estimate beta over a subsequent 36 month period prior to performing the Fama-MacBeth regressions. Therefore, we do not claim that traditional estimates of beta are never related to common stock returns. Instead, we show that the adjusted estimate of beta provides a better description of returns.

These results have an important practical implication. Beta services typically estimate betas for individual firms using about 60 months of historical data. Our results indicate that incorporating the information contained in past market returns improves the relationship between these firm-specific betas and future returns.

**Beta and the Size Effect**
The strong negative relationship between firm size and future returns is a well-known phenomenon. In this section we study the joint role of size and each estimate of beta in return.

To examine the beta and size relationship we use two additional sorting procedures which are common in the finance literature. First, we group firms into 20 portfolios based on market value of outstanding equity. Firm size is measured prior to the start of a given year. An equal-weighted return for each portfolio is calculated for each month of the following year. The portfolios are re-formed at the end of each year.

In an attempt to isolate size and beta effects, we also sort firms into 10 size groups, then firms in each size group are sorted by their historical $\beta$ or $SUM\beta$ estimate. An equal-weighted return for each portfolio is calculated for each month of the following year. The portfolios are re-formed at the end of each year. Therefore, this procedure yields 100 size-$\beta$ portfolios and 100 size-$SUM\beta$ portfolios. NYSE firms are used to determine size and beta breakpoints so that NASDAQ firms do not dominate the portfolios after 1972. Post ranking $\beta$ and $SUM\beta$ estimates are calculated over the entire 1931 through 1994 time period.

The joint roles of size, $\beta$ and $SUM\beta$ in returns is examined within the context of an expanded Fama-MacBeth cross-sectional regression. That is, portfolio excess returns are regressed against various combinations of the variables hypothesized to explain returns - size and beta. Full period $\beta$ and $SUM\beta$ estimates are used in the cross-sectional regressions. The time-series means of the regression slopes are then used to test the hypothesis that the variables are priced. The results are reported in Table 3.

Panel A reports the results of the cross-sectional regressions when firms are sorted into 20 size-based portfolios. When used in isolation, both $\beta$ and $SUM\beta$ are positively related to returns, whereas firm size is negatively related to returns. When both $\beta$ and size are used as explanatory variables, only firm size is significantly related to returns. This is evidence of the
size-effect in common stock returns. Most interestingly, when \( \text{SUM}\beta \) and firm size are both included as independent variables, only \( \text{SUM}\beta \) is significantly related to returns. Note that the regression adjusted \( R^2 \) values are 0.4 when either \( \beta \) or \( \text{SUM}\beta \) are used in combination with firm size in the cross-sectional regressions. Taken together, these results suggest that the size effect in stock returns may simply be a proxy for the information contained in the lagged beta estimate \( \beta_{-1} \). So these results support our earlier results. Specifically, incorporating the information contained in past market returns improves beta estimates.

Panel B displays the results of the Fama-MacBeth regressions for the 100 size-\( \beta \) sorted portfolios, while Panel C displays the results for the 100 size-\( \text{SUM}\beta \) sorted portfolios. Again, \( \beta \), \( \text{SUM}\beta \), and firm size are significantly related to returns when used in isolation. When size is used in combination with either \( \beta \) or \( \text{SUM}\beta \), only firm size is significant. However, combining size with either measure of beta does increase the average regression adjusted \( R^2 \) by at least 9%. This suggests that beta does explain variation in returns that size does not explain.

When highly correlated variables, like size and beta, are included as independent variables in a regression, the variable that is measured with the least error is likely to appear to be the most significant (note that despite the sorting procedure the correlation coefficient between the natural logarithm of firm size and \( \text{SUM}\beta \) is 0.65). Size is measured without error. \( \beta \) and \( \text{SUM}\beta \) are estimated using historical data. In addition, when we calculate \( \text{SUM}\beta \) we are estimating two regression coefficients. Moreover, grouping firms into 100 distinct portfolios leads to portfolios that may contain a small number of firms (as few as 3 firms in one portfolio in 1931), especially prior to the inclusion of NASDAQ firms in the late 1970’s. Therefore, in an attempt to minimize the statistical noise associated with each coefficient in the \( \text{SUM}\beta \) calculation, we separate \( \text{SUM}\beta \) into its two components \( \beta \) and \( \beta_{-1} \). The results of this
regression indicate that the coefficient on lagged market returns $\beta_{-1}$ is positively related to returns. Therefore, these results also indicate that incorporating the information contained in past market returns into beta estimation at least partially explains the size effect in returns.

**Summary**

No commercial beta services provide estimates of systematic risk that account for the lagged price response of small firms to market wide information. Our results indicate that beta estimates for small firms are severely biased downwards. Traditional beta estimates are unrelated to future returns. However, adjusted estimates of beta display the positive risk-return tradeoff implied by the CAPM. In addition, adjusted beta estimates are capable of accounting for part of the size effect in stock returns. Based on these results, we recommend commercial beta services incorporate [via regression equation (2)] the information contained in prior market returns.
ENDNOTES

1 Ibbotson Associates will revise their BetaBook and Cost of Capital Quarterly publications to account for cross-autocorrelations in returns.

2 Boudoukh, Richardson, and Whitelaw [1994] show that the asymmetry observed in the cross-autocorrelations is consistent with a high (low) level of own autocorrelation for small (large) firms along with a high contemporaneous correlation between large and small firms.

3 For a thorough explanation see Mech [1993].

4 For examples, see Atchison, Butler, and Simonds [1987], Conrad and Kaul [1989], and Lo and MacKinlay [1990b].

5 For a detailed exposition of this argument see Brennan, Jegadeesh, and Swaminathan [1993].

6 Our adjusted beta is essentially a Dimson [1983] beta with no leading market return. We are able to drop the leading market return in the regression estimator because we use the market capitalization weighted NYSE/AMEX/NASDAQ index as our measure of market returns. The NYSE/AMEX/NASDAQ index is heavily weighted toward large firms which trade frequently. So the sample firms do not lead the market. Moreover, the NYSE/AMEX/NASDAQ index has no autocorrelation. Thus, we do not need to adjust our SUMβ for autocorrelation in the index as is the case with Scholes and William’s [1977] betas.

7 See Berk [1995] for a detailed explanation of this idea.

8 An important exception in practice is Value Line. Value Line estimates beta using weekly data. Our evidence suggests that incorporating a lagged market return is important when monthly data is used to estimate beta. Therefore, several lagged market returns are needed when weekly data is used to estimate beta. Incorporating additional lags introduces additional statistical noise into the estimation process.

In the academic literature Kothari, Shanken, and Sloan [1995] estimate beta using annual data. Their motivation was to obtain a more accurate estimate of beta. Our motivation is the same, however, we improve upon our estimate of beta by incorporating the information contained in lagged market returns. There is nothing wrong with estimating beta using annual data. However, to get a reasonable number of observations for the regression many years of data must be employed, e.g. 20 to 70 years. The implicit assumption in the regression is that beta remains constant over the entire sample period. To assume that beta would remain constant over several decades is restrictive for many practical applications. Using monthly data mitigates against this problem.

9 We equal-weight returns to maintain consistency with previous research.

10 For examples see Kothari, Shanken, and Sloan and Eugene F. Fama and Kenneth R. French [1992].
REFERENCES


Table 1  
**Beta Estimates for NYSE/AMEX/NASDAQ Size Portfolios:** 1926-1994

All NYSE ordinary common stocks are ranked by size (market value of outstanding equity) and then split into deciles. The largest companies are put into decile 1, while the smallest are put into decile 10. The capitalization for the largest company within each decile serves as the breakpoint for that decile. Ten size portfolios are constructed by placing NYSE/AMEX/NASDAQ securities into portfolios based on NYSE breakpoints. Monthly portfolio returns are market capitalization weighted averages of the individual firms within each portfolio. Breakpoints are re-balanced on the last day of trading in March, June, September, and December of each year. $R_f$ is the 30-day T-bill rate observed at the beginning of the month. $R - R_f$ is the excess return on the portfolio. $\beta$ is the slope coefficient from a regression of the excess return on a portfolio on the excess return on the market portfolio [see regression equation (1)]. The market capitalization weighted NYSE/AMEX/NASDAQ index is used as the market portfolio. $SUM\beta$ is the adjusted estimate of beta measured as the sum of the slopes in the regression of the portfolio return on the current and prior month’s market return [see regression equation (2)].

<table>
<thead>
<tr>
<th>Size Portfolio</th>
<th>$R - R_f$</th>
<th>$\beta$</th>
<th>$SUM\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.58</td>
<td>0.93</td>
<td>0.91*</td>
</tr>
<tr>
<td>2</td>
<td>0.73</td>
<td>1.09</td>
<td>1.09</td>
</tr>
<tr>
<td>3</td>
<td>0.78</td>
<td>1.15</td>
<td>1.16</td>
</tr>
<tr>
<td>4</td>
<td>0.82</td>
<td>1.20</td>
<td>1.24*</td>
</tr>
<tr>
<td>5</td>
<td>0.87</td>
<td>1.24</td>
<td>1.30*</td>
</tr>
<tr>
<td>6</td>
<td>0.88</td>
<td>1.27</td>
<td>1.36*</td>
</tr>
<tr>
<td>7</td>
<td>0.91</td>
<td>1.34</td>
<td>1.45*</td>
</tr>
<tr>
<td>8</td>
<td>0.95</td>
<td>1.39</td>
<td>1.57*</td>
</tr>
<tr>
<td>9</td>
<td>1.02</td>
<td>1.47</td>
<td>1.63*</td>
</tr>
<tr>
<td>10</td>
<td>1.31</td>
<td>1.57</td>
<td>1.84*</td>
</tr>
</tbody>
</table>

*Significantly different from $\beta$ at the 5% level.
Figure 1
Size Portfolio Betas

All NYSE ordinary common stocks are ranked by size (market value of outstanding equity) and then split into deciles. The largest companies are put into decile 1, while the smallest are put into decile 10. The capitalization for the largest company within each decile serves as the breakpoint for that decile. Ten size portfolios are constructed by placing NYSE/AMEX/NASDAQ securities into portfolios based on NYSE breakpoints. Monthly portfolio returns are market capitalization weighted averages of the individual firms within each portfolio. Breakpoints are re-balanced on the last day of trading in March, June, September, and December of each year. $\beta$ is the slope coefficient from a regression of the excess return on a portfolio on the excess return on the market portfolio [see regression equation (1)]. The market capitalization weighted NYSE/AMEX/NASDAQ index is used as the market portfolio. $SUM\beta$ is the adjusted estimate of beta measured as the sum of the slopes in the regression of the portfolio return on the current and prior month’s market return [see regression equation (2)].
Table 2
Average Time-series Slopes from Fama-MacBeth Cross-sectional Regressions

Beginning in January 1931, we estimate $\beta$ and $SUM\beta$ for NYSE/AMEX/NASDAQ common stocks using the prior 60 months of historical data. $\beta$ is the slope coefficient from a regression of the excess return on a stock on the excess return on the market portfolio [see regression equation (1)]. $SUM\beta$ is the adjusted estimate of beta measured as the sum of the slopes in the regression of a stock return on the current and prior month’s market return [see regression equation (2)]. The market capitalization weighted NYSE/AMEX/NASDAQ index is used as the market portfolio. In Panel A, the realized monthly excess returns for the individual securities are regressed against each beta estimate [see regression equation (3)]. The process is repeated through December 1994. The cross-sectional regression coefficients are then averaged through time. In Panel B, firms are classified into 20 portfolios based on their historical estimate of $\beta$ or $SUM\beta$. The 5 percent of firms with the lowest historical beta are put into portfolio 1 and the 5 percent of firms with the highest historical beta are put into portfolio 20. An equal-weighted return for each month of the following year is calculated for each portfolio. Beta estimates are updated at the end of each year and the portfolios are re-formed. Portfolio betas are then estimated using the entire series of post-ranking portfolio returns. Each month the 20 portfolio excess returns are regressed against the full-period portfolio beta. The cross-sectional regression coefficients are then averaged through time.

<table>
<thead>
<tr>
<th>Risk Measure</th>
<th>$\lambda_0$ (%)</th>
<th>$t(\lambda_0)$</th>
<th>$\lambda_1$ (%)</th>
<th>$t(\lambda_1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Individual Firms</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.76</td>
<td>5.73*</td>
<td>0.26</td>
<td>1.55</td>
</tr>
<tr>
<td>$SUM\beta$</td>
<td>0.75</td>
<td>5.38*</td>
<td>0.27</td>
<td>1.85*</td>
</tr>
<tr>
<td><strong>Panel B: Beta sorted Portfolios</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.60</td>
<td>3.98*</td>
<td>0.39</td>
<td>1.49</td>
</tr>
<tr>
<td>$SUM\beta$</td>
<td>0.55</td>
<td>3.62*</td>
<td>0.41</td>
<td>1.71*</td>
</tr>
</tbody>
</table>

*Significantly greater than zero at the 5% level.
Table 3
Average Time-series Slopes from Fama-MacBeth Cross-sectional Regressions:
Size and Beta

Beginning in January 1931, NYSE/AMEX/NASDAQ common stocks are sorted into 20 portfolios based on the previous year’s firm size. An equal-weighted portfolio return is calculated for each month of the year. The portfolios are re-formed each year. The full 1931 through 1994 time period is used to estimate a β and \(SUMβ\) for each portfolio. \(β\) is the slope coefficient from a regression of the excess return on a size portfolio on the excess return on the market portfolio [see regression equation (1)]. \(SUMβ\) is the adjusted estimate of beta measured as the sum of the slopes in the regression of a portfolio return on the current and prior month's market return [see regression equation (2)]. The market capitalization weighted NYSE/AMEX/NASDAQ index is used as the market portfolio. In each month, portfolio excess returns are regressed against various combinations of \(\ln(\text{size})\), \(β\), \(SUMβ\), and the coefficient on the lagged market return \(β_{-1}\). The process is repeated through December 1994. The cross-sectional regression coefficients are then averaged through time and reported in Panel A. In Panel B, the above analysis is performed on 100 size-β portfolios. \(β\) is calculated for each firm using 60 months of historical data. Firms are first classified into 10 size groups, then into 10 β groups within each size group. Panel C reports the results for 100 size-SUMβ portfolios. t-statistics are in parentheses.

<table>
<thead>
<tr>
<th>intercept(%)</th>
<th>(β)</th>
<th>(SUMβ)</th>
<th>(\ln(\text{size}))</th>
<th>(β_{-1})</th>
<th>Ave. adj. (R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.45</td>
<td>1.93</td>
<td>0.34</td>
<td></td>
<td></td>
<td>0.34</td>
</tr>
<tr>
<td>(-2.76)*</td>
<td>(3.76)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.74</td>
<td>1.30</td>
<td>0.36</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-2.02)*</td>
<td>(3.79)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.84</td>
<td>-0.18</td>
<td>0.35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.26)*</td>
<td>(-3.89)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.79</td>
<td>-0.89</td>
<td>0.40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.61)*</td>
<td>(-1.60)</td>
<td></td>
<td>(-3.64)*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.28</td>
<td>1.01</td>
<td>0.40</td>
<td>-0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.29)</td>
<td>(2.19)*</td>
<td></td>
<td>(-1.11)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel A: Size Portfolios

<table>
<thead>
<tr>
<th>intercept(%)</th>
<th>(β)</th>
<th>(SUMβ)</th>
<th>(\ln(\text{size}))</th>
<th>(β_{-1})</th>
<th>Ave. adj. (R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33</td>
<td>0.56</td>
<td>0.17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1.94)</td>
<td>(2.05)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.89</td>
<td>-0.18</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.34)*</td>
<td>(-3.97)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.69</td>
<td>0.13</td>
<td>0.25</td>
<td>-0.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.87)*</td>
<td>(0.54)</td>
<td></td>
<td>(-4.51)*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Size-β Portfolios

-continued on next page-
Panel C: Size- \textit{SUMβ} Portfolios

<table>
<thead>
<tr>
<th></th>
<th>( \text{SUMβ} )</th>
<th>( \beta )</th>
<th>( \text{SUMβ} )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.19</td>
<td>0.63</td>
<td>0.18</td>
<td>(1.05)</td>
<td>(2.58) *</td>
</tr>
<tr>
<td>2.92</td>
<td>-0.19</td>
<td>0.15</td>
<td>(4.40) *</td>
<td>(-4.03) *</td>
</tr>
<tr>
<td>2.71</td>
<td>0.12</td>
<td>0.24</td>
<td>(5.93)</td>
<td>(-4.61) *</td>
</tr>
<tr>
<td>2.17</td>
<td>0.04</td>
<td>-0.12</td>
<td>(6.05) *</td>
<td>1.19</td>
</tr>
</tbody>
</table>

*Significantly different from zero at the 5% level.