ETF Index Tracking Analysis Notebook
Methodology

Morningstar Methodology Paper
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Foreword

Tracking difference and tracking error are commonly used to assess the quality of index portfolio management. The ETF Index Tracking Analysis Notebook allows users to calculate tracking difference and tracking error for index ETFs as outlined in this methodology document. Our approach addresses common factors that could affect tracking calculations, namely, market holidays and missing data. This notebook provides users with standardized measures of tracking performance and facilitates peer group comparisons.
Tracking Difference

Description
Tracking difference measures the difference in the performance of an index exchange-traded fund and its benchmark index over a specific period. An annualized figure should be used when comparing ETFs over the specified period. While this calculation is simple—merely the difference between an index ETF’s annualized net asset value total return and the annualized total return of the fund’s index—it requires precise data. An appropriate benchmark index is to be used, and the calculations require that data are available for both the ETF and the index on the start and end dates of the period in question. Note that ETFs and their benchmark indexes might not both have data on a particular day. For example, an ETF’s NAV may not be calculated in the event of a local market holiday but the ETF’s benchmark index may cover a foreign market that was open for trading that same day, and the index’s value would update accordingly.

The ETF Index Tracking Analysis Notebook provides annualized tracking difference figures for the trailing one-, three-, five-, 10-year and since-inception periods.

Calculation
A daily tracking difference is calculated for each date when both the ETF and the benchmark index have available data. This is done by calculating the day-to-day return from the previous date for which both the ETF and benchmark index have available data for both the ETF and the benchmark index and taking the difference of the two returns. This approach addresses both the potential issues outlined above: (1) trading holiday differences between the ETF and the benchmark index, and (2) any case where data is unavailable for other reasons.

Tracking Difference calculation for one-, three-, five-, and 10-year periods
For each day where both ETF and benchmark index have available data, calculate:

Let \( R_{ETF,d} \) be the ETF return on day \( d \) for a given ETF from day \( d-1 \) (day \( d-1 \) is defined as the day prior to day \( d \) for which both the ETF and the benchmark index have available data). Let \( R_{index,d} \) be the user-selected benchmark index return for day \( d \) from day \( d-1 \).

Daily Tracking Difference for day \( d \):

\[
TD_d = R_{ETF,d} - R_{index,d}
\]
Since weekends can be assumed as nontrading days, for any weekday \( j \) in the user-selected time period, an \( N \)-year annualized tracking difference can be calculated as follows:

Calculate \( N \)-year annualized return on weekday \( j \) of the ETF \( R_{ETF,j,N} \) and the \( N \)-year annualized return on weekday \( j \) of the benchmark index \( R_{index,j,N} \):

\[
R_{ETF,j,N} = \left[ \prod_{i=k}^{j} (1 + R_{ETF,i}) \right]^{1/N} - 1
\]

\[
R_{index,j,N} = \left[ \prod_{i=k}^{j} (1 + R_{index,i}) \right]^{1/N} - 1
\]

Where \( k \) is the first weekday from \( N \) years ago where both the ETF and the benchmark index have available data for \( R_{ETF,k} \) and \( R_{index,k} \) to be calculated.

Take the difference of these two returns to arrive at the \( N \)-year annualized tracking difference for weekday \( j \):

\[
N \text{-year Annualized Tracking Difference for weekday } j:
\]

\[
TD_{j,N} = R_{ETF,j,N} - R_{index,j,N}
\]

Note 1: The product of daily returns is used here to calculate the annualized returns of the ETF and the index. An alternative, and mathematically identical, approach would be to compare the NAV/index value of day \( j \) to that of \( N \) years ago from day \( j \). The product approach is used here as the daily return values are to be used when calculating tracking error—see subsequent section for details.

Note 2: An \( N \)-year tracking difference would not be computable if the length of available data is less than \( N \) years.

**Tracking Difference calculation for the since-inception period**

Since-inception annualized tracking difference is calculated similarly as follows using 365.25 days in a year in the annualization factor:

Calculate since-inception annualized return on weekday \( j \) of the ETF \( R_{ETF,j,\text{Inception}} \) and the since-inception annualized return on weekday \( j \) of the benchmark index \( R_{index,j,\text{Inception}} \):

\[
R_{ETF,j,\text{Inception}} = \left[ \prod_{i=k}^{j} (1 + R_{ETF,i}) \right]^{1/(365.25 N)} - 1
\]

\[
R_{index,j,\text{Inception}} = \left[ \prod_{i=k}^{j} (1 + R_{index,i}) \right]^{1/(365.25 N)} - 1
\]
Where \( k \) is the first weekday from inception for which both the ETF and the benchmark index have available data for \( R_{ETF,k} \) and \( R_{index,k} \) to be calculated and \( M \) is the number of calendar days between \( j \) and the inception date (if data is unavailable on inception date, the beginning date (that is, day \( k-1 \)) for which \( R_{ETF,k} \) and \( R_{index,k} \) were derived from would be used to calculate \( M \)).

Take the difference of these two returns to arrive at the since-inception annualized tracking difference for weekday \( j \):

**Since-Inception Annualized Tracking Difference** for weekday \( j \):

\[
TD_{j,Inception} = R_{ETF,j,Inception} - R_{index,j,Inception}
\]
Tracking Error

Description
Tracking error is defined as the standard deviation of a fund’s excess returns. In the context of an index ETF, excess returns refer to the absolute difference between the fund’s performance and that of its benchmark, its tracking difference. Given the nature of standard deviation and the usually small tracking differences for many index ETFs, any holiday differences and/or missing data could have a meaningful impact on the tracking error calculation. As such, it is important to ensure that the tracking differences are calculated where data are available for both the ETF and the index at the chosen start and end points.

Tracking error can be calculated across different time periods (for example, one, three, five years, or longer) and with different frequencies of observations (for example, daily, weekly, monthly). These choices can yield different results. The ETF Index Tracking Analysis Notebook provides calculations of annualized tracking error over the trailing one, three, five, and 10 years, as well as since the ETF’s inception using either daily or monthly tracking differences.

Calculation

Daily Observations

N-year Annualized Tracking Error, using daily tracking differences, on weekday j in the user-selected time period, is arrived at taking the standard deviation of the daily tracking differences in the past N years and annualized with annualization factor:

$$TE_{j,N} = \text{Standard Deviation}(R_{ETF,i} - R_{index,i}) \times \sqrt{250}$$

Or, in daily tracking difference notation:

$$TE_{j,N} = \text{Standard Deviation}(TD_i) \times \sqrt{250}$$

Where i is each weekday between day j and N-years prior to day j where data is available for the daily tracking difference to be calculated, inclusive of day j.

The since-inception annualized tracking difference is calculated similarly as follows by adjusting the time period of returns:

Since-Inception Annualized Tracking Error, using daily tracking differences, on weekday j:

$$TE_{j,\text{Inception}} = \text{Standard Deviation}(R_{ETF,i} - R_{index,i}) \times \sqrt{250}$$

Or, in daily tracking difference notation:

$$TE_{j,\text{Inception}} = \text{Standard Deviation}(TD_i) \times \sqrt{250}$$
Where $i$ is each weekday between day $j$ and inception date where data is available for the daily tracking difference to be calculated, inclusive of day $j$. (Same as the tracking difference calculation, if data is unavailable on inception date, the beginning date (that is, day $k - 1$) for which $R_{ETF,k}$ and $R_{index,k}$ were derived from would be used to replace the inception date).

Note 3: 250 days is used as an annualization factor for the tracking error (based on daily tracking differences) calculation to estimate number of trading days in a year. While understanding that there are differences in the number of trading days across global markets, 250 is an estimate for the purpose of comparing the tracking error figures of ETFs across different markets consistently. Total calendar days in a year (that is, 365.25 days, or the respective 365/366 days for each year) is not used here as daily tracking difference is only available on trading days.

Monthly Observations
For each month $m$, calculate:

Let $R_{ETF,m}$ be the ETF return for month $m$ for a given ETF and let $R_{index,m}$ be the user-selected benchmark index return for month $m$.

$$R_{ETF,m} = \left[ \prod_{i=k'}^{j'} (1 + R_{ETF,i}) \right] - 1$$

$$R_{index,m} = \left[ \prod_{i=k'}^{j'} (1 + R_{index,i}) \right] - 1$$

Where $k'$ is the first weekday in month $m$ for which both the ETF and the benchmark index have available data to calculate $R_{ETF,k'}$ and $R_{index,k'}$ and $j'$ is the last weekday in month $m$ for which both the ETF and the benchmark index have available data to calculate $R_{ETF,j'}$ and $R_{index,j'}$.

Monthly Tracking Difference for month $m$

$$TD_{m} = R_{ETF,m} - R_{index,m}$$

N-year Annualized Tracking Error, using monthly tracking differences, on month $p$ in the user-selected time period, is arrived at taking the standard deviation of the monthly tracking differences in the past $N$ years and annualized with annualization factor:

$$TE_{p,N} = \text{Standard Deviation}(R_{ETF,i} - R_{index,i}) \times \sqrt{12}$$

Or, in monthly tracking difference notation:

$$TE_{p,N} = \text{Standard Deviation}(TD_{i}) \times \sqrt{12}$$

Where $i'$ is each month between month $p$ and $N$-years prior to month $p$ where monthly data is available for the monthly tracking difference to be calculated, inclusive of month $p$ (that is, $N \times 12$ monthly data points available).
Since-Inception Annualized Tracking Difference, using monthly tracking differences, on month $p$ is calculated similarly as follows by adjusting the time period of returns:

$$TEm_{p,\text{Inception}} = \text{Standard Deviation}(Rm_{ETF,i} - Rm_{index,i}) \times \sqrt{12}$$

Or, in monthly tracking difference notation:

$$TEm_{p,\text{Inception}} = \text{Standard Deviation}(TDm_{i}) \times \sqrt{12}$$

Where $i'$ is each month between month $p$ and first available month end from the inception date where monthly data is available for the monthly tracking difference to be calculated, both inclusive.

Note 4: 12 months is used as an annualization factor for the tracking error (based on monthly tracking differences) as there are 12 months in a year. Tracking error is not calculated for periods less than one year.
Limitations

Data Quality
The calculations in this notebook are only as good as the data. If the ETF NAV data or index values are missing or incorrect, tracking difference and tracking error calculations will be impacted.

The daily tracking difference calculation looks back to the last available value for both the NAV and the index. This approach is designed to avoid potential errors owing to differences in data availability between the ETF and the index on a given date. Here is an illustrative example:

<table>
<thead>
<tr>
<th>Date</th>
<th>ETF NAV</th>
<th>Index value</th>
<th>$R_{ETF,d}$ (%)</th>
<th>$R_{index,d}$ (%)</th>
<th>TD$_d$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/29/Y1</td>
<td>10.0000</td>
<td>100.0000</td>
<td>N/A (Insufficient data)</td>
<td>N/A (Insufficient data)</td>
<td>N/A</td>
</tr>
<tr>
<td>12/30/Y1</td>
<td>10.1995</td>
<td>102.0000</td>
<td>1.9950</td>
<td>2.0000</td>
<td>-0.0050</td>
</tr>
<tr>
<td>12/31/Y1</td>
<td>No data</td>
<td>102.5000</td>
<td>Not calculated (No ETF NAV)</td>
<td>Not calculated (No ETF NAV)</td>
<td>N/A</td>
</tr>
<tr>
<td>1/1/Y2</td>
<td>No data</td>
<td>No data</td>
<td>Not calculated (No ETF NAV/index value)</td>
<td>Not calculated (No ETF NAV/index value)</td>
<td>N/A</td>
</tr>
<tr>
<td>1/2/Y2</td>
<td>10.3444</td>
<td>103.4500</td>
<td>1.4207</td>
<td>1.4216</td>
<td>-0.0009</td>
</tr>
<tr>
<td>1/3/Y2</td>
<td>No data</td>
<td>No data</td>
<td>Not calculated (No ETF NAV/index value)</td>
<td>Not calculated (No ETF NAV/index value)</td>
<td>N/A</td>
</tr>
<tr>
<td>1/4/Y2</td>
<td>No data</td>
<td>No data</td>
<td>Not calculated (No ETF NAV/index value)</td>
<td>Not calculated (No ETF NAV/index value)</td>
<td>N/A</td>
</tr>
<tr>
<td>1/5/Y2</td>
<td>10.3435</td>
<td>No data</td>
<td>Not calculated (No index value)</td>
<td>Not calculated (No index value)</td>
<td>N/A</td>
</tr>
<tr>
<td>1/6/Y2</td>
<td>10.1344</td>
<td>101.3520</td>
<td>-2.0301</td>
<td>-2.0280</td>
<td>-0.0021</td>
</tr>
</tbody>
</table>

Note: ETF NAV/Index value for illustration purpose only
Choosing a Benchmark Index
This notebook allows users to select their own benchmark. An appropriate benchmark, along with the correct return type (that is, total return, net return), should be chosen in order to correctly reflect the tracking performance of the chosen ETF and its peers during the selected period.

Changing Benchmark Indexes
The notebook is designed for the user to input one benchmark index for comparison purposes. For this reason, if an included ETF tracked a different index or multiple different indexes during the selected period of calculation, the tracking error and tracking difference figures will not be accurate for the period(s) when the ETF tracked a different index or indexes. Users can address this issue by selecting the period during which the ETF tracked the selected index.