Introduction
The Morningstar Sustainability Preferences Portfolio Construction Tool creates a portfolio tracking investment policy model while allowing investors to express their unique environmental, social, and governance preferences as defined in terms of product involvement and impact score.

This paper lists down the extensive checks performed on the Sustainability Preferences Portfolio Construction Tool, which range from impulse response to testing derivative conditions. The objective of performing such tests is to check if the results are as expected given user preference. Each test performed on the tool gives us confidence about the accuracy of the engine and optimality of the result.

Key Takeaways
- The tool satisfies the first-order and second-order conditions for optimization.
- The optimized value is a global optimum point, and the tool does not get stuck in local optimum points.
- The tool moves in the expected direction with changing input data and investor preferences.

The test results presented in the following pages are generated using a sample lineup of 42 funds that represent a typical use case for the tool. The results for other datasets also hold and can be generated if required. Some of the results presented here cannot be exactly replicated, though the conclusion will remain the same as per our understanding.

List of Checks Performed
- Change in optimal impact score with change in investor preference.
- Change in asset allocation with change in benchmark score.
- Shock to Product involvement revenue of the lineup.
- Shock to impact score of a fund with the highest impact score of a sustainability theme.
- Comparing objective value at the optimized point with other feasible points.
- First-order and second-order conditions at the optimized point.
Objective Function Defined in the Optimization Subject to Constraints

In this optimization, we define multiple objectives that minimize tracking error and maximize the impact score of the portfolio. These objectives are converted into a single objective by using an inbuilt multiplier lambda that is derived in multiple stages of the optimization. Please refer to our detailed methodology document here for more details about the tool.

\[
\max P_{\text{impact metric}} \lambda_{IM} wI - P_{\text{market}} \lambda_{TE} (w - w_p)^T \Sigma (w - w_p) \quad \ldots \quad (1)
\]

where:
- \( w \): vector of fund weights
- \( w_p \): vector of asset class policy weights
- \( I \): impact metric scores vector normalized to \([0,1]\) scale
- \( P \): user preference(s)
- \( \lambda \): multiplier assigned to each component
- \( \Sigma \): covariance matrix
1) Change in Optimal Impact Score With Change in Investor Preference

The effect of a change in investor preference on the optimized portfolio impact score is observed in this test. Given that our function is convex (see Equation 1 above; tracking error is convex and the impact score is a linear function), we should see the impact score change in an expected direction with a change in user preference. Each user preference theme is mapped to one or more impact metrics. See more about impact metric here.

Investor preferences for various sustainability themes are shocked (one at a time) by reducing the theme's weight in the preference vector and renormalizing the remaining weights to maintain the total vector sum as 1. The optimal value(s) of the impact metric(s) associated with the shocked theme are observed.

Although only two cases are presented here, results for other preference themes also hold. Following are the test results after shocking two sustainability themes.

1. Climate Change
   A change in the optimal portfolio's climate action score (response) is observed with a change in preference for the climate change theme (stimulus). A reduction in preference for climate change causes a reduction in the optimal portfolio's climate action score. This behavior is in line with the expectation.

   The chart below shows preference levels for climate change under different scenarios on the horizontal axis (a lower number implies lower preference) and the climate action impact score of the optimized portfolio is shown on the y-axis.

![Exhibit 1 Impact of Change in Climate Change Preference on Climate Action Score](source: Morningstar Quantitative Research)
2. Improve Peoples' Lives

This sustainability theme is associated with two impact metrics, namely basic needs and human development. A similar non-increasing trend in impact score is observed with a reduction in preference, which is expected.

Exhibit 2 Relationship Between Change in Preference for 'Improve Peoples' Lives' on Basic Needs and Human Development Scores

Source: Morningstar Quantitative Research.
2) Change in Asset Allocation With Change in Benchmark Score

The optimizer is trying to construct a portfolio with an asset allocation in line with the benchmark investment policy subject to a given range (plus/minus 5% here). In this test, the asset allocation in the benchmark policy is changed, and the resulting change in the asset allocation of the optimized portfolio is observed. A constraint-adhering optimizer should create a portfolio wherein the change in asset allocation of the tracking portfolio follows the change in the benchmark policy.

Two such test cases are presented here: Stock (US) and Stock (Developed Markets ex-US). The allocation of these two asset classes in the benchmark policy is shocked individually, and the asset allocation in the optimal portfolio is observed. The x-axis represents allocation in the benchmark and the y-axis shows asset allocation in the optimized portfolio.

Exhibit 3 Impact of Change in Benchmark Asset Allocation on Asset Allocation of the Optimized Portfolio

Source: Morningstar Quantitative Research.
3) Shock to Product Involvement Revenue

In this test, the product involvement score related to one product category—for example, tobacco, palm oil, and so on—for all securities is shocked and the consequent change in the product involvement score of the optimized portfolio is observed. This shock is very straightforward—when all funds see an increase/decrease a product involvement, the optimized portfolio should also see an increase/decrease in product involvement. See more about product involvement methodology [here](#).

The following chart shows the product involvement score of a particular product in the optimized portfolio on the y-axis. The x-axis values are the factor by which the product involvement of all funds in the lineup is multiplied. Two test cases are presented here, fossil fuel and gambling.

**Exhibit 4 Impact of Change in Product Involvement Metrics of all Securities Eligible for Selection on Product Involvement Score of the Portfolio**

Source: Morningstar Quantitative Research.
4) Shock to Impact Score of a Fund With the Highest Impact Score of a Sustainability Theme

Here, the impact score of the fund with the highest impact score within a selected preference theme is shocked and the ensuing change in the fund's weight in the optimized portfolio is observed.

The optimal weight of the fund with the highest score in a preference theme should increase with an increase in its impact metrics associated with that theme. The intuition here is that the optimizer, to maximize the impact metrics score of the portfolio, should 1) select the fund with the highest impact metrics and 2) allocate increasingly to the fund as the impact metrics improve. In a situation where the fund with the highest impact score is not eligible for the portfolio (because of product involvement screening), the fund with the next highest impact score is selected until an eligible fund is found.

The test case here shocks two impact metrics: resource security and climate action. The x-axis shows a multiplicative factor used to increase the impact score of the fund, and the y-axis shows its weight in the optimized portfolio.

The jump in portfolio weight in the charts below is permissible with an increase in impact score of a fund if the change is monotonic. The occurrence of such a jump in portfolio weight is less likely to happen in a bigger universe set relative to a small universe set like in our test case.

Exhibit 5a Impact of Change in Impact Score of a Fund on its Weight in the Optimized Portfolio

Source: Morningstar Quantitative Research.
Exhibit 5b  Impact of Change in Impact Score of a Fund on Its Weight in the Optimized Portfolio

Source: Morningstar Quantitative Research.
5) Comparing Objective Value at the Optimized Point With Other Feasible Points

It is possible that the optimized point for the tool turns out to be a local optimum point, and we miss out on global optima. The objective of this exercise is to make sure that the optimized point is better than other feasible points in the universe.

To achieve this, the objective value of the optimized portfolio is compared with other feasible random portfolios here. A feasible space is constructed using a set of constraints and random samples are generated from it using the Billiard walk algorithm. This algorithm is chosen over others as it covers the universe uniformly much faster than the other available algorithms (namely hit and run).

The process is as follows: A feasible space is constructed based on constraints and around 1,000 samples are generated from it. The objective value is calculated at all these samples and compared with the objective value at the optimized point. After multiple iterations of sampling and comparison, the results show that that optimized portfolio has a higher objective value than all other feasible points.

**Exhibit 6 Objective Value at Random Samples vs. Objective Value at the Optimized Point**

![Exhibit 6 Objective Value at Random Samples vs. Objective Value at the Optimized Point](source: Morningstar Quantitative Research.)
6) First-Order and Second-Order Conditions

For any point \( x \) to be an optimal point, two necessary and sufficient conditions are required to be satisfied.

- **First-Order Condition**
  \[ \nabla F(x)'(y - x) \geq 0 \text{ for all } y \text{ in the feasible universe} \quad \ldots \quad (2) \]

Here
1. \( \nabla F(x)' \) – First-order derivative of the objective function at the optimized point \( x \)
2. \( y \) is any random point from the feasible space

In the following exhibit, we calculate the left-hand side of eq (2) at 1,000 randomly sampled feasible portfolios to demonstrate that the expression is greater than zero for all the samples. This partially confirms that the solution \( x \) is indeed optimal.

**Exhibit 7 Left-Hand Side of the First-Order Condition Above**

In any constraint maximization problem, the optimum point must satisfy the negative definiteness condition of the bordered Hessian, that is,

\[
\begin{bmatrix}
0 \\
A^T \\
H
\end{bmatrix} \leq 0
\]

Here “\( H \)” is the second-order derivative of the objective function at the optimum point and \( A \) is the first-order derivative matrix of the binding constraints. Matrix \( A \) is constructed using all binding constraints. The negative semidefinite bordered Hessian condition is satisfied in all our test cases.
This condition is similar to the negative second-order condition for any unconstraint maximization problem.

**Exhibit 8** Negative Second-Order Condition for an Unconstrained Maximization Problem

In our testing, all different cases show a negative semidefinite matrix that satisfies the second-order condition.

Source: Morningstar Quantitative Research.
Exhibit 9 Second-Order Matrix of the Objective Function (H)

Source: Morningstar Quantitative Research.
Exhibit 10 First-Order Matrix of the Binding Constraints (A)

Source: Morningstar Quantitative Research.
Conclusion
The Morningstar Sustainability Preferences Portfolio Construction Tool is being tested from multiple dimensions before rolling it out to users. The results so far indicate robust performance under many different possible scenarios.

The impulse response indicates that the optimizer is moving in the expected direction as data/preference changes, in line with user expectations. Users would expect the output to change in an intuitive direction with changes in their preferences. Additionally, the derivative conditions ensure that the optimized point is truly an optimal point from the theoretical point of view.

The sampling exercise is our attempt to visually show a regular user (one who is not aware of the mathematical conditions required for the optimized point) that the optimized point is best among the feasible samples.

This optimization is our first such engine, and we will continue to add more testing features. When we add any new feature, we intend to do similar testing before rolling out the feature.
About Morningstar® Quantitative Research
Morningstar Quantitative Research is dedicated to developing innovative statistical models and data points, including the Morningstar Quantitative Rating, the Quantitative Equity Ratings and the Global Risk Model.

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